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## ENERGY LEVELS IN BAND SPECTRA

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### ABSTRACT

While the radiating potentials of monatomic gases can usually be definitely connected with lines of the series spectra of the elements, those of diatomic and multiatomic gases frequently cannot. This has led several investigators to suggest that the potentials are for the molecule, but they have not been connected with the band spectra of the substances. The connection of these radiating potentials with the band spectra is pointed out in this paper.

A brief synopsis of the present methods of analysis of band spectra is given, and energy-level diagrams illustrating typical electronic and vibrational shifts are shown and discussed. Two types of grouping of bands are illustrated. The normal state of the molecule is that of the zero electronic and vibrational states coupled with some rotational state  $m$ . Consequently, the first band system should consist of a single set of bands whose frequencies are related in a second Deslandres progression, but succeeding systems should consist of groups of bands. The extent of a band system is limited by the overlapping of energy levels, by the stability of the molecule, and by the ionization of the molecule. The minimum radiating potential of a system is the potential of the zero vibrational level. The radiating potentials may be experimentally determined by photographing the spectra excited by electron impacts of increasing energy. References are made to experiments of this kind. The zero level of the ionized molecule is discussed.

While the experimentally determined radiating potentials of monatomic substances can usually be definitely connected with lines of the series spectra of the elements, the critical potentials of diatomic and multiatomic gases very frequently cannot. Often, when the frequency of a line corresponding to a certain radiating potential is computed from the  $eV = h\nu$  relation, no line of this frequency can be found in the spectrum of the element. This has

led several experimenters to suggest in such cases that the critical potential is for the molecule and not for the atom. Attempts have been made in a few cases to distinguish between the critical potentials of the atom and those of the molecule, but no attempts have been made to relate the radiating potentials of the molecule with its band spectrum. It is the purpose of this paper to show what that relationship is and to demonstrate how the connection can be made.

#### ANALYSIS OF BAND SPECTRA

Recent progress in the analysis of band spectra enables us to determine the connection between the spectrum and the critical potentials of a molecule. The structure of a band spectrum indicates that there are, in addition to a change in its translational motion, three ways by which a molecule may absorb energy instead of only one, as in the case of the atom. These three processes have been associated with changes in the electronic configuration of the molecule, in the configuration of the nuclei of the constituent atoms, and in the rotational state of the molecule. The idea of stationary states is carried over to the molecule, and these changes are said to result in electronic shifts, vibrational shifts, and rotational shifts, respectively.

The amount of energy gained or lost by a molecule in an electronic shift is usually considerably greater than that of a vibrational shift, which, in turn, is considerably greater than that of a rotational shift. The amount of energy involved in the last is very small, so that, if the only change in the molecule is in its rotational state, the frequency of the radiation associated with such a change is very low, and the purely rotational spectra are found in the far infra-red. The change in energy during simultaneous rotational and vibrational shifts is somewhat greater but is still so small that the rotational-vibrational spectra are also in the infra-red. When electronic shifts occur, however, the amount of energy absorbed or radiated is considerably greater, and the spectrum may be in the visible or in the near or extreme ultra-violet. It is this third type of band spectrum that we shall be concerned with in this paper, as the methods ordinarily employed to measure the critical potentials of a substance are not applicable for very small changes of energy.

When the change in the state of a molecule involves a change in two or all three kinds of shifts simultaneously, it is assumed that the frequency of the radiation associated with the change is determined by the total change in the energy of the molecule such that

$$E' - E = h\nu, \quad (1)$$

where  $E'$  and  $E$  are the amounts of energy in the initial and final states, respectively. Not much is known as yet concerning the possible electronic shifts, but no selective principle is apparent in the vibrational transitions. On the other hand, rotational shifts are limited to those for which the rotational quantum number changes by  $\pm 1$  or  $0$ . These limitations are the cause of the three series,  $R$ ,  $P$ , and  $Q$ , into which the lines of a band are classified. Each type of transition contributes a share to the frequency of the emitted line, and a band system of the simplest type can be summed up in the following frequency-term expressions:

$$\left. \begin{aligned} P(m) &= F(e', n', m-1) - f(e, n, m) \\ Q(m) &= F(e', n', m) - f(e, n, m) \\ R(m) &= F(e', n', m+1) - f(e, n, m) \end{aligned} \right\} \quad (2)$$

where  $e'$  and  $e$  designate the initial and final electronic states, respectively;  $n'$  and  $n$  are the quantum numbers of the initial and final vibrational states, and  $m$  is the rotational quantum number of the final state. Thus a band system comprises those bands which are associated with a common electronic transition.

Present methods of analysis of band spectra enable us to determine the contributions of the several transitions to the frequencies of the lines of a band. The contributions of the rotational transitions can be determined by methods based upon the suggestions of Schwarzschild, Bjerrum, and Lenz, and outlined by Sommerfeld.<sup>1</sup> Such an analysis results in a classification of the lines with respect to the rotational states and in the determination of the position of the null line. Heurlinger<sup>2</sup> has shown that the frequencies of the null lines are related in a formula which can be written:

$$\nu = \nu_0 + (a'n' + b'n'^2) - (an + bn^2). \quad (3)$$

<sup>1</sup> *Atombau und Spektrallinien*, chap. vii.

<sup>2</sup> *Zeitschrift für Physik*, **1**, 82, 1920.

This formula is the natural successor to Deslandres' "Law of the Edges of Bands," and enables us to determine the contributions of the vibrational transitions to the frequencies of the lines of a band. Thus far the contributions of the electronic shifts have not been subjected to any law. They can be determined only by subtracting the contributions of the rotational and vibrational shifts from the total energy represented by the emission of a line. The task of analyzing the bands and band systems of known spectra has only been begun, and so one can hardly say how successfully the energy states of a molecule can be determined from such analyses. The classification of the lines of bands is proceeding more rapidly and in some respects more satisfactorily than the classifications of the bands of a system. There are so many adjustable constants in equation (3) that different investigators report quite different results from an analysis of the same system.

Equation (3), however, successfully accounts for the main features of a band system. If the difference in vibrational numbers,  $n' - n$ , is fixed, the value of  $\nu$  will change a little for the successive values of  $n$  (or  $n'$ ). As a consequence, there result neighboring bands for each value of  $n' - n$  that occurs, and this accounts for the grouping of the bands that is so prominent a feature of many band systems. One group represents the shift  $n' - n = 0$ , and the several bands are due to the transitions  $0 \rightarrow 0$ ,  $1 \rightarrow 1$ ,  $2 \rightarrow 2$ , etc. The next group in ascending frequency scale will be due to the shift  $n' - n = 1$ , and the bands of the group represent the transitions  $1 \rightarrow 0$ ,  $2 \rightarrow 1$ , etc., while the next group in descending frequency scale will result from the shift  $n' - n = -1$ , viz.,  $0 \rightarrow 1$ ,  $1 \rightarrow 2$ , etc. Figure 2 represents a system with this type of grouping. If  $n'$  is constant and  $n$  varies, one gets a set of bands whose frequencies form approximately an arithmetic series. This relationship is expressed as a first Deslandres progression, and such bands have common initial states. If  $n$  is constant and  $n'$  varies, the resulting set have frequencies expressed as a second Deslandres progression. Obviously, if all the possible values of  $n'$  and  $n$  are taken, both progressions represent the same system.

Another type of grouping will result if the vibrational levels of the initial states are close together compared with those of the final



states or vice versa. Figure 3 shows three groups of such a system. If the initial vibrational levels are close together, as illustrated in the diagram, the bands of each group are related in a second Deslandres progression, whereas, if the final states had been the closer, each group would have formed a first Deslandres progression.

#### THE NORMAL STATE OF A MOLECULE

The normal state of a molecule is that in which the energy associated with the electronic and nuclear configurations is a minimum. Consideration of the specific heats and observations on the infra-red absorption spectra of gases indicate that practically all of the molecules are in the same, zero, vibrational state and in the zero electronic state. A certain percentage of the molecules may be expected to be in the lower-quantum vibrational states at ordinary temperatures, and this percentage will increase with the temperature because of the greater energy of thermal agitation. A condition of statistical equilibrium will obtain at any given temperature such that a certain percentage of the molecules will be found in each vibrational state. The energy of most molecules in vibrational state 1 is so great compared with the average kinetic energy that only a small percentage is found in any except the zero vibrational state. For the rotational states, however, we have a different condition. The amount of energy involved in a rotational shift is small, comparable with the energy of thermal agitation at ordinary temperatures, and so we may expect to find the molecules in various states of rotation. Specific heat considerations indicate that the molecules are, in general, rotating, and infra-red absorption spectra show that they are distributed among the possible rotational states according to some distribution law. We can, then, designate the normal state of a molecule as the  $o, o, m$  state, i.e., the zero electronic and vibrational states and some rotational state  $m$ . When a molecule absorbs energy it may change in any one or all three of the ways possible to it, depending upon the amount of energy absorbed and the manner in which it is communicated to the molecule. The result is that the molecule is left in some higher stationary (excited) state in which it will remain for a longer or shorter time.

## ENERGY-LEVEL DIAGRAM

We can represent the possible stationary states of a molecule by means of a ladder diagram similar to that sometimes employed in line spectra. We will take as our base line the  $o, o, o$  state, and, in order to simplify the diagram, will construct only the zero rotational levels. Hence Figures 2 and 3 show only the null lines of the bands. In one type of grouping the bands for which  $n' - n = \Delta n = 0$  will form one group, those for which  $\Delta n = 1$  another group, etc. These groups may be separated or may overlap, depending on the constants in equation (3). Figure 2 shows the groups of bands that make up the system.

The grosser characteristics of a band system of this type are readily deduced from the diagrams. If a molecule has only five possible vibrational states, as in the hypothetical case represented (Fig. 2), the system would have one group of five bands and two each of four, three, two, and one. The five group is that for which  $\Delta n = 0$  and has the  $o \rightarrow o$  band at its head. Systems of this type are often observed with a number of groups containing the same number of bands, but a diminution in the number in the end-groups is common. One reason why a number of groups may have the same number of bands may lie in the probabilities of the transitions. Usually, bands originating from and terminating at the lower-quantum states are the more probable. Likewise, bands for which  $\Delta n$  is small are more probable than those for larger vibrational shifts. In the case represented, the initial levels are farther apart than the final levels. If the reverse were true, the order of the bands in the groups would be reversed.

If the normal state of a molecule is the  $o, o, m$  state, Figure 1 shows that the first band system should consist of a single set of bands related in a second Deslandres progression. Unfortunately, the first system of the gases investigated lies in the extreme ultra-violet in all cases, and so this point cannot be checked at present. From the foregoing, however, this must be true at least for absorption bands of homopolar molecules. It will be interesting to learn whether it is possible to have a more extensive system of emission bands. If so, either there are vibrational-rotational bands in emission that do not ordinarily occur in absorption, or else there are

stationary states from which the molecule cannot return to the normal state—a circumstance that sometimes occurs in atoms. In this first system the  $o \rightarrow o$  band will be the leading band and will have the lowest frequency. All systems except the first should show grouping. This grouping may be pronounced or scarcely apparent,

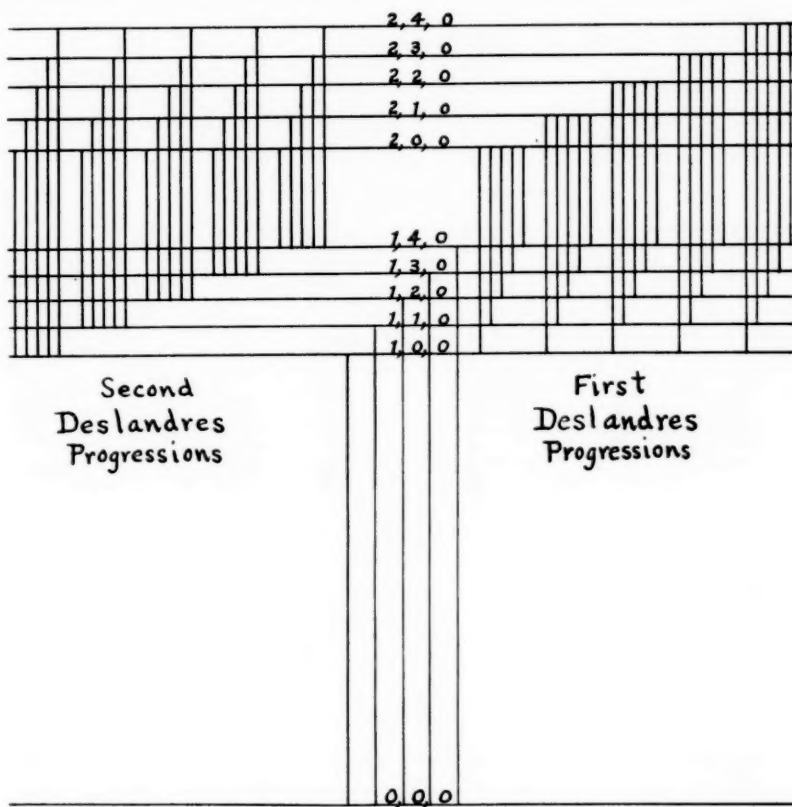


FIG. 1

depending on the binding forces in the molecule and the way in which these change with the electronic shifts.

Figures 1 and 2 do not show an altogether hypothetical case. The vibrational levels shown are a part of those drawn by Birge<sup>1</sup> for the first positive bands of nitrogen, and the diagrams are scaled to

<sup>1</sup> *Physical Review*, 23, 294, 1924.

show a few of the bands of this system. This system has been measured only to the band designated  $0 \rightarrow 0$ ,  $\lambda 9108 \text{ \AA}$ .<sup>1</sup> The diagram shows a few of the bands of longer wave-length that must exist to complete the system. The system probably extends with diminishing intensity to several  $\mu$  in the infra-red. The group for  $\Delta n = -1$  should lie between  $1.05 \mu$  and  $0.97 \mu$ , and can be photographed by dyeing the plates.

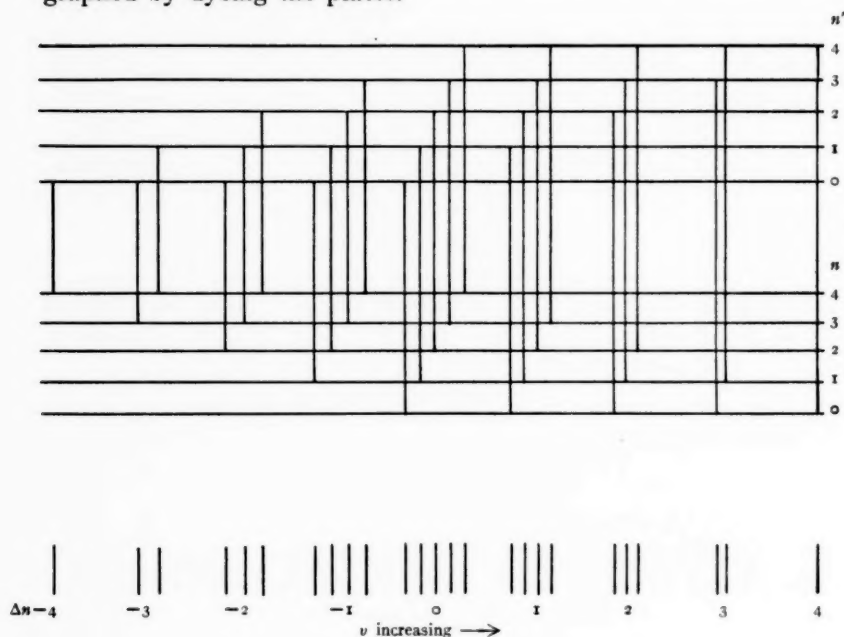


FIG. 2

Figure 3 represents a part of a band system with groups of the second type. In this type of grouping, each group will have the same number of bands with relative intensities depending on the relative probabilities of the states. Duncan suggests that the fourth positive bands of nitrogen are a system of this type. The entire system should appear at once if the bands are excited by electron impacts of increasing energy, for the difference in energy of the initial states is far too small to permit the successive development of

<sup>1</sup> Birge, *Astrophysical Journal*, **39**, 50, 1914; Croze, *Annales de Physique* (9) **1**, 35, 1914.

the bands. These bands were found by D. C. Duncan<sup>1</sup> to appear simultaneously when excited in this way. Figure 3 is scaled to represent a part of this system.

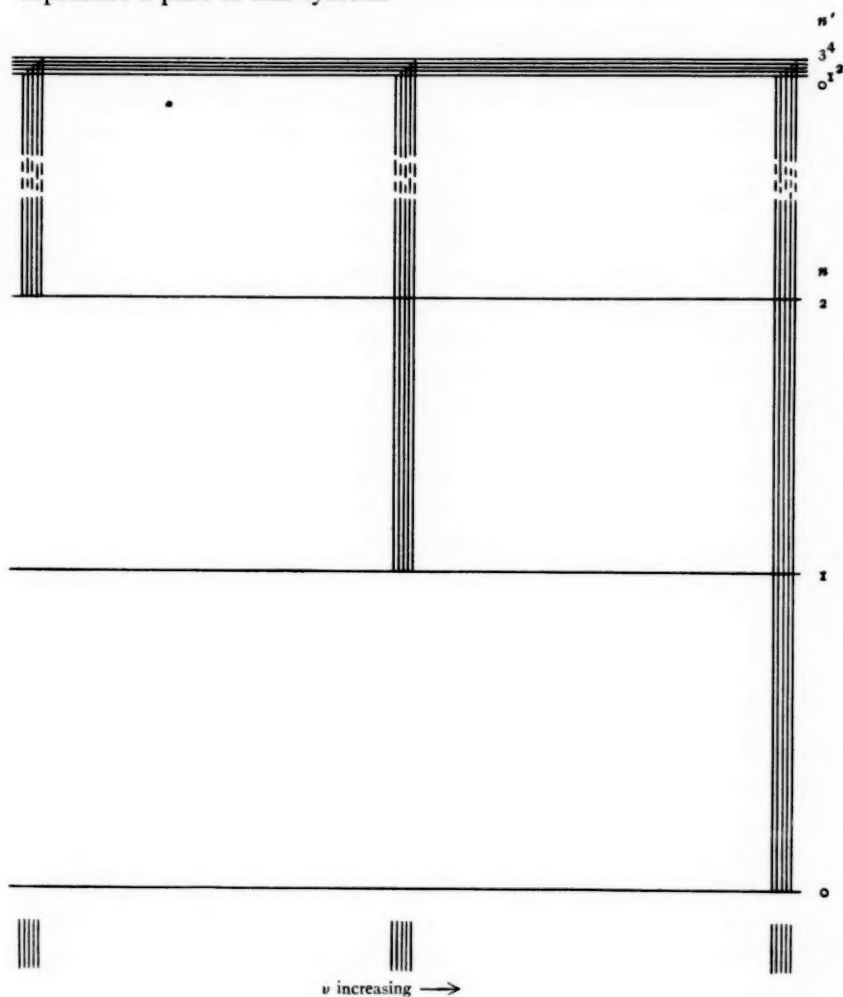


FIG. 3

Bands belonging to different systems are easily distinguished from one another in most cases. Band systems are usually dis-

<sup>1</sup> Thesis, University of Michigan, 1924. (To be published in *Astrophysical Journal*.)

tinctly separated and frequently have great differences in the structure and spacing of the bands and in other physical characteristics. Overlapping of systems sometimes occurs, but is never very pronounced. Thus, if the molecule represented in Figure 1 has an electronic state 3, such that the energy in the shift  $3, n', m' \rightarrow 2, o, m$  of maximum energy change is less than the energy change in the shift,  $2, o, m' \rightarrow 1, n, m$  of minimum energy change, the systems will overlap. This occurs when one system possesses high vibrational quantum states. But the probability of shifts from or to states of high quantum number is small, and so the overlapping bands will be weak and the overlapping not very extensive. Overlapping would also occur if the energy level of the  $3, o, m$  state were lower than that of  $2, n, m$ , and the electronic shift  $3 \rightarrow 1$  were possible. In this case, the probability of a molecule's being in a vibrational state of small quantum number coupled with the electronic state 3 would probably be greater than that of its being in a vibrational state of large quantum number coupled with electronic state 2, and so, again, the overlapping would be limited. The band system for the electronic shift  $2 \rightarrow 1$  would gradually give way to that for electronic shifts  $3 \rightarrow 1$  and  $3 \rightarrow 2$  as the energy of excitation is increased. Such a process is suggested by the changes in the intensities of the positive band systems of nitrogen observed by Duncan.<sup>1</sup>

#### FACTORS LIMITING BAND SYSTEMS

Other factors limit the extent of a band system besides the overlapping of energy levels. The stability of the molecule sets a limit to the amount of energy that can be absorbed by a nuclear shift. One would expect a relationship between the heat of dissociation of a gas and the extent of its vibrational states. It is a matter of common observation that the most stable molecules have extensive band systems, but no intimate relation has yet been worked out. The ionization of the molecule is an upper limit for the electronic shifts. If the radiating potential for the  $o \rightarrow o$  band of a system is near the ionizing potential, the system will probably not be well developed, for as soon as the ionizing potential is reached, it becomes a question of relative probabilities as to whether the molecule will

<sup>1</sup> Thesis, University of Michigan, 1924. (To be published in *Astrophysical Journal*.)



be ionized or put into a highly excited state. The probability of ionization quickly passes that of a high electronic and vibrational state after the ionizing potential is exceeded, and so the bands originating in high quantum states are weak or fail altogether. This is suggested as the cause of the poor development of the fourth positive band system of nitrogen.

#### RADIATING POTENTIALS OF A MOLECULE

It will be seen at once from Figure 1 that the radiating potentials of a molecule are less sharply defined than those of an atom. Radiation will set in as soon as the energy absorbed is sufficient to produce the 1, 0, *m* state, and will continue during the development of the system. When the 2, 0, *m* level has been reached, new radiation will set in and again at the 3, 0, *m* level. The voltage at which a strong system begins is what is usually detected by the methods that have been employed to measure radiating potentials. This is the potential of the initial state of the 0→*n* bands of the system and not necessarily that of the band of lowest frequency. If the system detected is that associated with the zero electronic end-state, the radiating potential should agree with

$$V = \frac{h\nu_0}{e},$$

where  $\nu_0$  is the frequency of the null line of the 0→0 band. Brandt<sup>1</sup> has succeeded in detecting the successive appearance of bands or band series having the initial vibrational states 0, 1, 2, etc. It might be well to reserve the name "radiating potential" for that of the 0→0 band of each system. If this is done, relationships can be established between the radiating potentials and the electronic states of a molecule.

#### EXPERIMENTAL DETERMINATION OF ENERGY LEVELS

Investigations during the past two years by the writer and his associates<sup>2</sup> show that the development of band systems, when they are excited by electron impacts of increasing energy, is consistent

<sup>1</sup> *Zeitschrift für Physik*, **8**, 32, 1921.

<sup>2</sup> Duffendack, "The Excitation of the Secondary Spectrum of Hydrogen by Electron Impacts," *Astrophysical Journal*, **60**, 122, 1924; Duncan, *loc. cit.* (nitrogen); Lockrow, in preparation (oxygen).

with the foregoing. Bands originating in the zero vibrational state appear first and a characteristic development outward from these bands follows. The voltage at which bands could first be photographed by a long exposure was taken as the excitation potential of the system. Obviously, the voltage determined in this way is subject to error on account of variations in the sensitivity of the instruments used, but this error is quite small, for it was found in all cases that at slightly higher voltages the bands could be photographed easily and new members appeared. Spectrographs of high speed were employed, and usually check exposures of ten to sixteen hours were made at slightly less voltages. After the first bands appeared, exposures were at first made at intervals of 1 volt, but Lockrow has found that he can show the development of one of the oxygen systems in quarter-volt intervals. No difficulty has been experienced in separating one system from another. The energy differences of the electronic shifts are great enough to permit the successive development of the band systems of all the substances investigated thus far.

Reference to the detailed reports of the investigations completed will show the relationship between the energy levels determined by this method and the radiating potentials that have been reported for hydrogen, nitrogen, and oxygen. The emission of the Fulcher bands of hydrogen leaves the molecule with a potential energy of about 11 volts. A band system in the extreme ultra-violet must exist, the emission of which returns the molecule to the normal state. The radiating potential of about 11 volts that has been reported for hydrogen<sup>1</sup> marks the setting in of this system. In nitrogen two strong systems end at a level of about 8.2 volts, and so a system in the region of 1500 Å must return the molecule to the normal state. Bands have been reported in this region by Lyman,<sup>2</sup> one of the strongest of which is at 1550 Å. The analysis

<sup>1</sup> Franck, Knipping, and Kruger, *Verhandlungen der deut. phys. Gesells.*, **21**, 728, 1919; Davis and Goucher, *Physical Review*, **10**, 101, 1917; Horton and Davies, *Royal Society Proceedings, A*, **97**, 23, 1920; Horton and Davies, *Philosophical Magazine*, **46**, 872, 1923; Compton and Olmsted, *Physical Review*, **17**, 45, 1921; Foote, Mohler, and Kurth, *ibid.*, **19**, 414, 1922.

<sup>2</sup> *Spectroscopy of the Extreme Ultra-violet*.

of this system should show its initial states to be common with the end-states of the two systems ending at this level. Several observers have reported a radiating potential for nitrogen at about 8.2 volts.<sup>1</sup> The measurement and analysis of band systems in the extreme ultra-violet is greatly desired in connection with the analysis of systems of less frequency.

The determination of the zero level of the ionized molecule is presenting some unexpected difficulties. It was naturally supposed that the ionizing potential would represent the zero level, and the relationship of enhanced band systems of oxygen fit in nicely

TABLE I

Pfund	Measured	$n' \rightarrow n$	$\Delta n$
0.59 $\mu$ .....	.6186	4 $\rightarrow$ 1	3
0.68.....	.6968	2 $\rightarrow$ 0	2
0.78.....	.7889	1 $\rightarrow$ 0	1
0.90.....	.9108	0 $\rightarrow$ 0	0
	Computed	$n' \rightarrow n$	$\Delta n$
1.06.....	1.051	0 $\rightarrow$ 1	-1
1.20.....	1.236	0 $\rightarrow$ 2	-2
.....	1.497	0 $\rightarrow$ 3	-3

with this hypothesis. In nitrogen, the first and second negative bands appear simultaneously at about 18.5 volts. The difference between this and the ionizing potential is very nearly the energy emitted in the 0  $\rightarrow$  0 band of the first system, but the emission of the second negative bands leaves the molecule with considerably less energy than that represented by the ionizing potential. Whether it is possible to have an energy level for the ion below the ionizing potential remains to be seen. This would mean that a considerable change in the configuration of the nuclei is possible after ionization.

NOTE.—Pfund's paper in the September number of the *Journal of the Optical Society* came into my hands the day after I had com-

<sup>1</sup> Davis and Goucher, *Physical Review*, **13**, 1, 1919; Smyth, *ibid.*, **14**, 409, 1919; Mohler and Foote, *Journal of the Optical Society of America*, **4**, 49, 1920; Brandt, *loc. cit.*; Boucher, *Physical Review*, **19**, 189, 1922.

pleted this manuscript. He reports finding nitrogen bands out to  $1.2 \mu$  by means of an infra-red spectrometer in good agreement with those found by Coblentz.<sup>1</sup> These bands are undoubtedly a part of the first positive system, and are probably the leading bands of the groups indicated in Table I. The first four bands listed were classified by Birge,<sup>2</sup> and his classification is extended to the others. The measured wave-lengths are those of Birge and Croze.<sup>3</sup>

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<sup>1</sup> *Investigations of Infra-red Spectra*, **1**, 6, 317.

<sup>2</sup> *Physical Review*, **23**, 294, 1924.

<sup>3</sup> Birge, *Astrophysical Journal*, **39**, 50, 1914; Croze, *Annales de Physique* (9), **1**, 35, 1914.

## A LIST OF ULTIMATE AND PENULTIMATE LINES OF ASTROPHYSICAL INTEREST<sup>1</sup>

By HENRY NORRIS RUSSELL

### ABSTRACT

*Spectral lines of astrophysical interest.*—The very rapid development of the analysis of complex line-spectra makes it desirable to prepare a preliminary list of lines of astrophysical interest for which the spectroscopic and energy-relations are known. A brief summary of the theory is given, followed by a list of lines, the elements being arranged in order of the successive groups of the periodic table, and the individual lines in order of the energy-level of origin within the atom. A bibliography of the more important recent papers is appended.

*Terminology.*—The name *ultimate lines* is suggested for those lines which originate in the lowest energy-level (or normal state) of an atom (whether neutral or ionized) or in another state very close to this. Lines originating in neighboring but higher energy-levels may be called *penultimate lines*; those originating in still higher levels, *ante-penultimate lines*. Ultimate lines, as here defined, are nearly, but not quite, synonymous with the *raies ultimes* of De Gramont. The difference is discussed in the text. The meaning of the terms "flame," "arc," and "spark lines" is also discussed. An index to the table has been made in which the elements are arranged in the alphabetical order of their chemical symbols.

The great advances which have recently been made in the interpretation of line-spectra, and the astrophysical importance of the results, appear to justify a collection from the widely scattered sources of the material of most importance in astrophysical work.

1. *Summary of the theory.*—It is now generally accepted that the emission of a given spectral line depends upon a transition of the emitting atom between two definite states (fixed by quantum conditions) in which its internal energy is different, and that the number of waves per second (or per centimeter) in the resulting radiation is exactly proportional to the energy released by the atom. In absorption this process is reversed.

The familiar spectroscopic terms, whose differences give the wave-numbers of the various lines, therefore indicate the *energy-levels* of the corresponding atomic states, which may thus be mapped out with extreme accuracy. According to Bohr, each term corresponds to a different arrangement of electron orbits within the atom, and for terms belonging to the same series the orbital angular momentum of the outermost electron is the same. Series for which

<sup>1</sup> Contributions from the Mount Wilson Observatory, No. 286.

this differs by a unit may be called "adjacent." Most terms are multiple and represent several closely neighboring energy-levels, differing probably in the arrangement of inner orbits. They may thus be grouped into systems, each containing several series, characterized by a certain multiplicity or maximum number of components (which, however, is not attained in all terms).<sup>1</sup> Three or more systems of different multiplicity may occur in the same spectrum. In such cases they are all of odd or all of even multiplicity, according as the atomic number is even or odd (the only known exception occurring in helium).

The spectrum of an ionized atom (enhanced lines) is in these respects similar to that of the element of the next preceding atomic number: that of a doubly ionized atom, to the element preceding this, and so on.

Transitions accompanied by radiations or "combinations" occur normally between terms belonging to adjacent series in the same system. Which components combine with one another is determined by the "inner-quantum" rules.<sup>2</sup> Such a combination forms a "multiplet," or group of lines, which may number from 1 to 16.

In the more complex spectra, combinations between terms belonging to the same series occur, and a few between terms of non-adjacent series, and also between terms of different systems. The last two classes usually give faint lines.

2. *Levels of origin.* *Ultimate, penultimate, and antepenultimate lines.*—In astrophysical applications, these somewhat complex series relations are less important than the simpler matter of the energy-level of origin of the lines, by which is meant the lower of the two levels involved in the corresponding transition. An atom cannot absorb the line unless it is in this state.

The relative numbers of atoms which, under equilibrium conditions, will be in the various energy-levels, are determined by general thermodynamic considerations.<sup>3</sup> At low temperatures, practically all the atoms will be in the state of least internal energy, and capable of absorbing only lines which correspond to transitions from this

<sup>1</sup> For details see Landé, *Zeitschrift für Physik*, **15**, 189, 1923.

<sup>2</sup> Sommerfeld, *Atombau und Spektrallinien*, 1922.

<sup>3</sup> Fowler and Milne, *Monthly Notices*, **83**, 403, 1923.



state to others. These lines alone will be found in the absorption spectrum of the cool gas, or its emission spectrum at a slightly higher temperature. As the temperature is raised, a small but ever increasing proportion of the atoms will, at any given instant, be in states of higher energy-content, and the lines originating in these will appear in the order of their levels of origin, and increase in relative intensity with rising temperature. Comparison of the spectroscopic data with King's temperature classification shows, in all cases which have been investigated, a beautifully perfect correlation, which puts the correctness of this interpretation beyond doubt.

Lines originating in the lowest level are, for obvious reasons, prone to reversal, and are usually the last lines to disappear when only traces of the element are present. The latter are De Gramont's *raies ultimes*. Following his lead, we may call all lines originating in the lowest energy-level "ultimate lines." For those originating in the next lowest level, we may borrow a good word from the grammarians and call them "penultimate lines"; while lines arising from still higher levels may be termed "antepenultimate."

If the two lowest levels are very near together (as in  $Ti^+$ ), lines arising from both may be called ultimate, and two or more nearby levels may similarly be included as origins for penultimate lines.

Enhanced lines may be classified in the same way. Ultimate lines, arising from the lowest energy-level in the ionized atom, will be absorbed as soon as the gas begins to be ionized, and will therefore appear even in the furnace, and be strong in the arc, like H and K. Antepenultimate enhanced lines, like  $Mg^+\lambda 4481$ , which require that the ionized atom shall receive a large additional input of energy before it can absorb them, and still more before it can emit them, may not appear in the arc at all.

It should be added that ultimate lines, in the sense here defined, are not always identical with De Gramont's *raies ultimes*. The latter denote the most persistent lines in the region of the spectrum which is easily observable, and, if this includes no lines originating in the lowest level, lines arising from the lowest available level will be chosen, so that the *raies ultimes* may be penultimate lines (for example, the green magnesium triplet for visual observations).

Conversely, ultimate lines, though originating in the lowest level, are sometimes faint, and disappear when the concentration is small, so that they are not recorded as *raies ultimes*. For example, the calcium line  $\lambda$  4226.7 (1S-1P) is a typical *raie ultime*, while  $\lambda$  6572.7 (1S-1p<sup>3</sup>), an intersystem combination, is of moderate intensity; and  $\lambda$  4575.4 (1S-1D), a combination between non-adjacent series, is a very faint line. All these are true ultimate lines, greatly strengthened in the furnace at low temperatures, in sun-spots, and in dwarf stars.

There would appear to be no reason why the French phrase should not be retained in use, as at present, with the significance attached to it by its author, while a somewhat different meaning is given to the English words.

In the spectra first investigated (the alkalis and alkaline earths), the lowest energy-level (or normal state) of the atom is 1S, and the ultimate lines belong to the principal series 1S-mp; but this is not always the case. In aluminium, for example, the normal state is 1p<sup>2</sup>; in iron, 1d<sup>5</sup>; and in titanium, 1f<sup>3</sup>. The principal series in these spectra, when they have been detected at all, are inconspicuous, and the ultimate lines belong, in different cases, to very different series, many of which have no specific names.

3. *Ionization and excitation potentials*.—As the vapor of a given element is heated, the ultimate arc lines appear first, followed by the penultimate and antepenultimate. In due time ionization begins; the ultimate enhanced lines appear, and later the penultimate, while the arc lines fade, disappearing in the inverse order of their appearance, since, even at high temperatures, many more neutral atoms are in the normal state than in any other. Still farther on, the enhanced lines give place to lines corresponding to a higher degree of ionization, or often apparently to nothing at all, if the latter lie far in the ultra-violet.

In a normal atmosphere, however, various elements will go through this process at very different rates, determined by the ionization and excitation potentials, which measure the energy required to ionize an atom, starting from its normal state, or to raise it from this state to the various energy-levels from which penultimate and antepenultimate lines originate. This energy is usually

expressed in volts, one "volt" being the energy acquired by an electron which has fallen through this potential difference, and equalling 8100 of the frequency units used to express the spectroscopic terms.

These data bid fair to furnish the basis for a quantitative theory of stellar spectra, permitting the calculation of pressures and temperatures in the atmosphere.<sup>1</sup> Qualitatively, the order of appearance and disappearance of the lines of various elements in the stellar sequence may be understood by mere inspection of the data.

In Table I data are collected for those elements for which information is at present available, or can be readily derived from published data. Fowler's memoir<sup>2</sup> is the principal source, supplemented as indicated in the references. The lines of each element are grouped according to their excitation potentials (E.P.), the ultimate lines coming first. The wave-lengths are given in I.A.; the intensities and temperature classes taken from King when possible. Intensities from other sources are placed in parentheses. Following this, in the fifth column, is the series notation showing between what terms the transition occurs (the lower coming first). The successive series of terms are denoted, as usual, by the letters "s," "p," "d," "f," "g," "h," "i," and the system to which they belong by an exponent representing the multiplicity—singlet terms being represented, as usual, by capitals. The successive terms of a series are numbered consecutively, starting with the lowest, which is usually numbered 1. Fowler's notation has been followed for lines given by him.

Terms for which the combining properties are peculiar, and which do not belong to the regular series, are denoted by accents as "p'," "p'". Lines of different systems are listed separately. Distinctions between the components of multiple terms, which are unimportant for the present purpose, are ignored, and but a single-series designation given for a whole multiplet (except in one or two cases, such as *Tl*, where the separations are very wide). The ionization potentials, I.P., and the excitation potentials, E.P., for penultimate and other lines, are given when known.

<sup>1</sup> Fowler and Milne, *loc. cit.*

<sup>2</sup> *Report on Series in Line Spectra*. London: Fleetway Press, 1922.

The spectra of ionized atoms are listed independently, as are those of multiply ionized atoms in the few cases that have been investigated. As the list has been prepared for astrophysical purposes, only lines between  $\lambda$  3000 and  $\lambda$  8500 are listed except for important ultimate and penultimate lines. Only the strongest lines are given for elements like *Hg*, which do not appear in the solar spectrum; while the lists for astrophysically important elements like *Ca* and *Ti* are extended. The fainter components of multiplet groups are given only in the region where they may be observed in the sun-spot spectrum. Cases where lines are omitted are denoted by an asterisk under the series notation. Inter-system combinations, and those between non-adjacent series, are placed in parentheses.

As an illustration of the notation, the calcium line  $\lambda$  6572.78 ( $1S-1P^3$ ) is seen to be an intersystem combination line between a singlet level,  $1S$ , and a triplet level,  $1P$ . Since it is an ultimate line,  $1S$  must be the normal state of the atom, and since the excitation potential is less for the lines arising from  $1P$  than for any others, this is the next lowest energy-state, and the lowest in the triplet system.

4. *Flame, arc, and spark lines.*—The data which are summarized in Table I throw a flood of light on many spectroscopic problems.

Consider, for example, the relation of flame (or furnace), arc, and spark spectra. In the first of these, when the temperature is low, only ultimate lines appear (as in the Bunsen-flame spectra of the alkalis and alkaline earths (neglecting band-spectra due to compounds). For refractory elements, such as iron or titanium, the temperature of volatilization is usually high enough to bring out the penultimate lines in addition; though, when the spectrum is obtained at the very lowest practicable temperature, the ultimate lines may appear alone, as King has shown in the case of the emission spectrum of iron<sup>1</sup> and Grotrian for several absorption spectra.<sup>2</sup> When the ultimate lines of the neutral atom lie far in the ultra-violet, and the excitation potential for the penultimate lines is high

<sup>1</sup> *Mt. Wilson Contr.*, No. 247; *Astrophysical Journal*, 56, 370, 1922.

<sup>2</sup> *Zeitschrift für Physik*, 18, 169, 1923.

(as in hydrogen or oxygen), no observable flame or furnace spectrum is found.

Under the strong excitation of the arc all the lines of the neutral atom appear, and, since ionization is necessary for the passage of the current, lines of the ionized atom are also present. The ultimate lines of this second spectrum are always strong in the arc (unless, indeed, they lie in regions inaccessible to observation, as is presumably the case with the alkalis), and, if the ionization potential is low, they may appear in the furnace, even at moderate temperatures (as in *Ca*, *Sc*, *Ti*, etc.). Penultimate enhanced lines of low excitation potential appear faintly in the furnace, and are fairly strong in the arc. With increasing E.P. such lines become harder to get, and antepenultimate lines, such as  $Mg^+ \lambda 4481$ , cannot be produced in the ordinary arc at all, though the corresponding ultimate lines are strong.

In the spark, the lines of the ionized atom are conspicuous (and those of the multiply ionized atom also, if they are accessible), while the lines of the neutral atom also appear, since the atoms ionized by the discharge must return to normal. Enhancement, and not mere presence, in the spark is therefore the practical test.

If, however, the ionization and excitation potentials for a line of the neutral atom are both high (as in the case of the red triplet of oxygen), these lines may actually appear stronger in the spark than the arc, the excitation in the former being too weak for them.<sup>1</sup> The degree of enhancement for the lines of the ionized atom is, however, so much greater that there is usually little difficulty in deciding which are the true enhanced lines.

The use of the familiar and convenient terms "flame" or "furnace" lines for the ultimate lines of the neutral atom (or penultimate lines of low E.P.), "arc" lines for those of the neutral atom in general, and "spark" or "enhanced" lines for those of the ionized atom meets with no real difficulties, so long as it is remembered that the visible arc lines of one element may demand a higher degree of excitation than the enhanced lines of another.

The table also explains the great astrophysical importance of

<sup>1</sup> Merrill, *Mt. Wilson Contr.*, No. 183, p. 7; *Astrophysical Journal*, 51, 242, 1920.

the elements in the first half of the first long period of the periodic table—from *Ca* to *Fe*. Ultimate lines of all these elements lie in the easily observable region, and are usually numerous, while penultimate lines, usually of low excitation potential, abound. Their ionization potentials are also moderately low, and fairly similar, so that their enhanced lines are fairly prominent in the sun and conspicuous in stars of class F, but disappear above class A, owing to double ionization.

For the lighter elements of the two short periods, the ultimate lines are in the far ultra-violet (except for six lines belonging to *Li*, *Na*, *Al*, and *Mg*), and the excitation potentials of the penultimate lines are, in general, high. For this very reason, as well as their high ionization potentials, elements of this group furnish the characteristic lines in the spectra of classes A and B.

The prominence of scandium, titanium, and vanadium in the sun-spot spectrum follows at once from their low ionization potentials, and the great number of their ultimate and penultimate lines in the region which is best observable.

The analysis of the more complex spectra is at present advancing very rapidly. Among those not yet interpreted, the most important astrophysically are *Si*, *Co*, *Ni*, *Nb*, *La*, *Ce*, and among spark spectra  $C^{++}$ ,  $N^+$ ,  $O^+$ ,  $Si^+$ ,  $Si^{++}$ ,  $V^+$ ,  $Cr^+$ ,  $Mn^+$ . Almost all of these are known to be already under investigation, and some of the others (notably  $Fe^+$ ) are being studied at this observatory. The present list will therefore require considerable extension in a year or two. It is planned to use it here in studies of solar, sun-spot, and stellar spectra, and it is published in its present incomplete form in the hope that it may be of use to other workers in astrophysics.

The writer takes pleasure in expressing his obligation to Drs. Meggers, Kiess, and Walters for their generous communication of valuable data in advance of publication and to Miss Charlotte E. Moore for much help in tabulating and checking the data.



TABLE I

E.P.	$\lambda$	Int.	Class	Type	Notes
Hydrogen, H, I.P. 13.54					
0.0.....	1215.68 1025.83	..... .....	..... .....	1S-2P 1S-3P	
10.15.....	6562.79 4861.33 4340.46 4101.74 3970.08 3889.05 3835.39 3797.90 3770.63 3750.15	(10) (9) (8) (7) (6) (5) (4) (3) (2) (1)	..... ..... ..... ..... ..... ..... ..... ..... ..... .....	2P-3D (H $\alpha$ ) 2P-4D (H $\beta$ ) 2P-5D (H $\gamma$ ) 2P-6D (H $\delta$ ) 2P-7D (H $\epsilon$ ) 2P-8D (H $\zeta$ ) 2P-9D (H $\eta$ ) 2P-10D (H $\theta$ ) 2P-11D (H $\iota$ ) 2P-12D (H $\kappa$ )	
12.63.....	18751.05 12818.11	(10) (5)	..... .....	3D-4F 3D-5F	
Helium, He, I.P. 24.41 (1)					
0.0.....	591.5 584.4 557.1	..... ..... .....	..... ..... .....	(oS-1p <sup>2</sup> ) oS-1P oS-1P	
19.68.....	10830.30 10829.09 3888.65 3187.74	(200) (10) (8)	..... ..... ..... .....	1S <sup>2</sup> -1P <sup>2</sup> 1S <sup>2</sup> -2P <sup>2</sup> 1S <sup>2</sup> -3P <sup>2</sup>	
20.49.....	20582.04 5015.68 3964.72 3613.64 3447.59	(20) (6) (4) (3) (2)	..... ..... ..... ..... .....	1S-1P 1S-2P 1S-3P 1S-4P 1S-5P	
20.81.....	7065.68 7065.19 5875.96 5875.61 4713.37 4713.14 4471.69 4471.48 4120.98 4120.81 4026.36 4026.19 3867.62 3867.46 3819.75 3819.61 3705.13 3705.00	(1) (5) (1) (10) (1) (3) (1) (6) (1) (3) (1) (5) (1) (2) (1) (4) (1) (3)	..... ..... ..... ..... ..... ..... ..... ..... ..... ..... ..... ..... ..... ..... ..... ..... ..... .....	1P <sup>2</sup> -2S <sup>2</sup> 1P <sup>2</sup> -2d <sup>2</sup> (D <sub>3</sub> ) 1P <sup>2</sup> -3S <sup>2</sup> 1P <sup>2</sup> -3d <sup>2</sup> 1P <sup>2</sup> -4S <sup>2</sup> 1P <sup>2</sup> -4d <sup>2</sup> 1P <sup>2</sup> -5S <sup>2</sup> 1P <sup>2</sup> -5d <sup>2</sup> 1P <sup>2</sup> -6d <sup>2</sup>	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Helium—Continued					
21.12.....	7281.35	(3)	.....	1P-2S	
	6678.15	(6)	.....	1P-2D	
	5047.74	(2)	.....	1P-3S	
	4921.93	(4)	.....	1P-3D	
	4437.55	(1)	.....	1P-4S	
	4387.93	(3)	.....	1P-4D	
	4143.77	(2)	.....	1P-5D	
	4009.27	(1)	.....	1P-6D	
Ionized Helium, $He^+$ , I.P. 54.18					
0.0.....	303.8	.....	.....	1S-2P	
40.63.....	1640.49	.....	.....	2P-3D	
	1215.18	.....	.....	2P-4D	
48.16.....	4685.81	.....	.....	3D-4F	
	3203.16	.....	.....	3D-5F	
50.79.....	6560.16	.....	.....	4F-6G	
	5411.57	.....	.....	4F-7G	
	4859.36	.....	.....	4F-8G	
	4541.63	.....	.....	4F-9G	
	4338.71	.....	.....	4F-10G	
	4199.87	.....	.....	4F-11G	
	4100.08	.....	.....	4F-12G	
	4025.64	.....	.....	4F-13G	
	3968.47	.....	.....	4F-14G	
	3923.51	.....	.....	4F-15G	
Neon, $Ne$ , I.P. 21.5 (2)					
16.56( $s_4$ ) to 16.78 ( $s_2$ ) ..	6598.95	(15)	.....	1S <sub>4</sub> -2P <sub>2</sub>	
	6506.53	(15)	.....	1S <sub>4</sub> -2P <sub>8</sub>	
	6402.25	(20)	.....	1S <sub>5</sub> -2P <sub>9</sub>	
	6382.99	(12)	.....	1S <sub>4</sub> -2P <sub>7</sub>	
	6266.49	(15)	.....	1S <sub>3</sub> -2P <sub>5</sub>	
	6217.28	(15)	.....	1S <sub>5</sub> -2P <sub>7</sub>	
	6163.59	(12)	.....	1S <sub>3</sub> -2P <sub>2</sub>	
	6143.06	(12)	.....	1S <sub>5</sub> -2P <sub>6</sub>	
	5975.53	(12)	.....	1S <sub>5</sub> -2P <sub>5</sub>	
	5881.90	(20)	.....	1S <sub>5</sub> -2P <sub>2</sub>	
	5852.49	(50)	.....	1S <sub>2</sub> -2P <sub>1</sub>	
	5400.56	(50)	.....	1S <sub>4</sub> -2P <sub>1</sub>	
	3593.52	(10)	.....	1S <sub>2</sub> -3P <sub>4</sub>	
	3520.47	(20)	.....	1S <sub>2</sub> -3P <sub>1</sub>	
	3472.57	(10)	.....	1S <sub>5</sub> -3P <sub>9</sub>	
	3417.90	(10)	.....	1S <sub>4</sub> -3P <sub>4</sub>	
	3369.91	(15)	.....	1S <sub>5</sub> -3P <sub>2</sub>	
	3369.81	(10)	.....	1S <sub>5</sub> -3P <sub>4</sub>	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Neon—Continued					
18.3 to 18.5	5764.43	(15)	.....	$2p_9-4d'_4$	
	5343.30	(12)	.....	$2p_{10}-4d_6$	
	5341.10	(20)	.....	$2p_{10}-4d_5$	
	5330.79	(12)	.....	$2p_{10}-4d_3$	
	4788.93	(12)	.....	$2p_9-5s_5$	
	4715.34	(15)	.....	$2p_9-6d'_4$	
	4708.86	(12)	.....	$2p_{10}-5d_5$	
	4704.39	(15)	.....	$2p_{10}-5d_3$	
Lithium, Li, I.P. 5.37					
0.0.....	6707.85	(10R)	.....	$1s^2-1p^2$	
	3232.61	(8R)	.....	$1s^2-2p^2$	
1.84.....	8126.52	(10)	.....	$1p^2-2s^2$	
	6103.53	(10R)	.....	$1p^2-2d^2$	
	4971.93	(7r)	.....	$1p^2-3s^2$	
	4602.90	(9R)	.....	$1p^2-3d^2$	
	4273.28	(5r)	.....	$1p^2-4s^2$	
	4132.29	(8)	.....	$1p^2-4d^2$	
Sodium, Na, I.P. 5.12					
0.0.....	5895.93	(10R)	.....	$1s^2-1p^2 (D_1)$	
	5889.96	(8R)	.....	$(D_2)$	
	3427.1	(1)	.....	$(1s^2-2d^2)$	
	3302.94	(8R)	.....	$1s^2-2p^2$	
	3302.34	(7R)	.....		
2.09.....	8194.82	(10R)	.....	$1p^2-2d^2$	
	8183.30	(8R)	.....		
	6160.72	(8r)	.....	$1p^2-3s^2$	
	6154.21	(8r)	.....		
	5688.22	(10)	.....	$1p^2-3d^2$	
	5682.67	(8)	.....		
	5153.64	(6n)	.....	$1p^2-4s^2$	
	5149.00	(5n)	.....		
	4982.87	(6r)	.....	$1p^2-4d^2$	
	4978.61	(5r)	.....		
Potassium, K, I.P. 4.32					
0.0.....	7699.01	(10R)	.....	$1s^2-1p^2$	
	7664.94	(10R)	.....		
	4642.17	(2)	.....	$(1s^2-2d^2)$	
	4641.58	(1)	.....		
	4047.20	(6R)	.....	$1s^2-2p^2$	
	4044.14	(8R)	.....		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Potassium—Continued					
1.61.....	6938.98	(8)}	.....	$1p^2-3s^2$	
	6911.30	(7)}			
	5832.31	(7R)}	.....	$1p^2-4d^2$	
	5812.71	(6R)}			
	5802.16	(6R)}	.....	$1p^2-4s^2$	
5782.77	(5R)}				
Rubidium, <i>Rb</i> , I.P. 4.16					
0.0.....	7947.64	(8R)}	.....	$1s^2-1p^2$	
	7800.29	(10R)}			
	4215.56	(7R)}	.....	$1s^2-2p^2$	
	4201.82	(8R)}			
1.58.....	7757.83	(8)}	.....	$1p^2-3d^2$	
	7619.12	(8)}			
	7408.37	(10)}	.....	$1p^2-3s^2$	
	7280.22	(10)}			
Caesium, <i>Cs</i> , I.P. 3.88					
0.0.....	8943.46	(10R)}	.....	$1s^2-1p^2$	
	8521.12	(10R)}			
	4593.16	(6)}	.....	$1s^2-2p^2$	
	4555.26	(8)}			
1.45.....	9172.23	(2)}	.....	$1p^2-3d^2$	
	8761.25	(5)}			
	7944.11	(6)}	.....	$1p^2-3s^2$	
	7609.13	(5)}			
1.71.....	8079.24	(8r)}	.....	$2d^2-4f^2$	
	8015.90	(8r)}			
Copper, <i>Cu</i> , I.P. 7.69					
0.0.....	3273.97	(10R)}	.....	$1s^2-1p^2$	
	3247.55	(10R)}			
1.64.....	5782.16	(8)}	.....	$x-1p^2$	
	5700.25	(6)}			
3.80.....	8092.76	(10)}	.....	$1p^2-2s^2$	
	7933.19	(10)}			
	5220.04	(6)}	.....	$1p^2-2d^2$	
	5218.17	(10)}			
	5153.23	(8n)}			
	4530.84	(6r)}	.....	$1p^2-3s^2$	
	4480.38	(6r)}			
	4063.30	(4n)}	.....	$1p^2-3d^2$	
	4062.69	(6n)}			
	4022.67	(6n)}			

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Silver, <i>Ag</i> , I.P. 7.54					
0.0.....	3382.86 3280.66	(10R) (10R)	.....	1s <sup>2</sup> —1p <sup>2</sup>	
3.86.....	8273.73 7688.12	(10) (10)	.....	1p <sup>2</sup> —2s <sup>2</sup>	
	5471.51 5465.45 5209.08	(6) (10R) (10R)	.....	1p <sup>2</sup> —2d <sup>2</sup>	
	4668.52 4476.12	(8r) (6r)	.....	1p <sup>2</sup> —3s <sup>2</sup>	
	4210.71 4055.31	(8R) (6R)	.....	1p <sup>2</sup> —3d <sup>2</sup> ★	
Gold, <i>Au</i> , I.P. 9.20 (3)					
0.0.....	2675.95 2427.98	(10R) (10R)	.....	1s <sup>2</sup> —2p <sup>2</sup>	
4.60 } 5.08 } .....	7510.74 5837.43 4811.63 4792.60 4065.07	(6) (6) (5) (8) (6)	.....	2p <sup>2</sup> —2s <sup>2</sup> 2p <sup>2</sup> —2d <sup>2</sup>	
Beryllium, <i>Be</i> , I.P. 7.5: (4) (5)					
0.0.....	2348.63	(10)	.....	1S—1P	
? .....	3321.35 3321.00 3321.01	(8) (8)	.....	1p <sup>2</sup> —1s <sup>3</sup>	
5.21.....	4572.69	(9)	.....	1P—1D	
Ionized Beryllium, <i>Be</i> <sup>+</sup> , I.P. ?					
0.0.....	3131.07 3130.42	(12) (12)	.....	1s <sup>2</sup> —1p <sup>2</sup>	
Magnesium, <i>Mg</i> , I.P. 7.61					
0.0.....	4571.15 2852.11	5 300R	I A III	(1S—1p <sup>2</sup> ) 1S—1P	
2.67.....	5183.67 5172.70 5167.38	125 80 40	II II II	1p <sup>3</sup> —1s <sup>3</sup>	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes	
Magnesium—Continued						
2.67—Cont.	3838.29	100r	II	1p <sup>3</sup> —2d <sup>3</sup>		
	3832.31	80r	II			
	3829.36	40	II			
	3336.69	20	III	1p <sup>3</sup> —2s <sup>3</sup>		
	3332.14	15	III			
	3329.93	10	III			
4.33.....	8806.75	(8)	.....	1P—2D		
	5711.13	1	V	1P—3S		
	5528.42	10	V	1P—3D		
	4703.00	40	V	1P—4D		
	4351.91	30	IV	1P—5D		
	4167.39	10n	III?	1P—6D		
	4057.63	5n	III	1P—7D		
	Ionized Magnesium, Mg <sup>+</sup> , I.P. 14.97					
0.0.....	2802.70	(50)	IV	1s <sup>2</sup> —1p <sup>2</sup>		
	2795.52	(50)	IV			
4.41.....	2936.50	(35)	IV	1p <sup>2</sup> —2s <sup>2</sup>		
	2928.63	(35)				
	2797.99	(40)	.....	1p <sup>2</sup> —2d <sup>2</sup>		
	2790.77	(40)				
8.42.....	3615.64	(3)	.....	2s <sup>2</sup> —3p <sup>2</sup>		
	3613.80	(4)				
8.83.....	4481.33	(100)	.....	2d <sup>2</sup> —3f <sup>2</sup>		
	4481.13	....				
	3104.81	(30)	.....	2d <sup>2</sup> —4f <sup>2</sup>		
	3104.71	....				
9.95.....	4433.99	(8)	.....	2p <sup>2</sup> —4s <sup>2</sup>		
	4427.99	(7)				
	4390.58	(10)	.....	2p <sup>2</sup> —4d <sup>2</sup>		
	4384.64	(8)				
Calcium, Ca, I.P. 6.09 (6)						
0.0.....	6572.78	8	I A	(1S—1p <sup>3</sup> )		
	4575.43	0	.....	(1S—1D)		
	4226.73	500R	I	1S—1P		
1.88.....	6162.18	150	II	1p <sup>3</sup> —1s <sup>3</sup>		
	6122.22	100	II			
	6102.72	80	II			
	4456.61	10	II	1p <sup>3</sup> —2d <sup>3</sup>		
	4455.88	40	I			
	4454.77	80	I			
	4435.67	40	I			
	4434.95	60r	I			
	4425.43	50	I			



TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Calcium—Continued					
1.88—Cont.	4318.65	45	I	$1p^3-1p'^3$	
	4307.74	45	I		
	4302.52	60r	I		
	4298.99	30	I		
	4289.38	40	I		
	4283.01	40	I		
	3973.72	12	III	$1p^3-2s^3$	
	3957.05	10	III		
	3948.90	6	III		
	3644.40	40	III	$1p^3-3d^3$	
	3630.75	30	III		
	3624.11	20	III		
2.53.....	6508.84	(1)	..	$1d^3-1f''^3$	
	6499.65	30	II		
	6493.79	80	II		
	6471.66	40	II		
	6462.58	125	II		
	6439.09	150	II		
	6464.70	(1)	..	$(1d^3-1D')$	
	6455.61	10	II		
	6449.81	50	II		
	6169.58	40	III	$1d^3-2p^3$	
	6169.03	25	III		
	6166.44	15	III		
	6163.75	10	III		
	6161.31	10	III		
	6156.08	(0)	III		
	5602.83	20	III	$1d^3-1d'^3$	
	5601.28	30	III		
	5598.48	50	III		
	5594.46	60	III		
	5590.11	20	III		
	5588.75	80	III		
	5581.97	25	III	$1d^3-1p''^3$	
	5270.27	60	III		
	5265.56	40	III		
	5264.24	20	III		
	5262.23	25	III		
	5261.70	20	III		
	5260.38	2	III	$1d^3-3f^3$	
	4585.87	50	II		
	4581.41	40	II		
	4578.57	30	II	$1d^3-4f^3$	
	4098.55	15	III		
	4094.94	12	III		
	4092.65	8	III	$(1D-1f'^3)$	
2.70.....	7202.16	3	II		
	7148.12	10	II		
	6717.69	30	III	$1D-2P$	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Calcium—Continued					
2.70—Cont.	5349.47	25	III	1D—1F'	
	5041.61	40	III	1D—3P	
	4878.13	50	III	1D—3F	
	4526.94	30	III	1D—4P	
	4355.10	25	III	1D—4F	
	4240.46	6	III	1D—5P	
	4108.55	10N	III	1D—5F	
	3972.58	3	III	1D—6F	
2.92.....	7326.10	2	III	1P—2D	
	5857.48	100	III	1P—1P'	
	5512.98	20n	III	1P—3s	
	5188.85	50	III	1P—3D	
	4847.29	2	III A	1P—4s	
	4685.26	12	III	1P—4D	
Ionized Calcium, $\text{Ca}^+$ , I.P. 11.82					
0.0.....	3968.47	350R	II	1S <sup>2</sup> —1P <sup>2</sup> (H)	
	3933.66	400R	II	(K)	
1.71.....	8662.11	(9)	V	1d <sup>2</sup> —1p <sup>2</sup>	
	8542.15	(10)	V		
	8498.00	(8)	V		
3.14.....	3736.90	12	V	1p <sup>2</sup> —2s <sup>2</sup>	
	3706.02	10	V		
	3181.28	4	V	1p <sup>2</sup> —2d <sup>2</sup>	
	3179.34	15	V		
	3158.88	10	V		
Strontium, Sr, I.P. 5.67 (6)					
0.0.....	6892.62	50r	I	(1S—1P <sup>3</sup> )	
	4607.34	600R	I	1S—1P	
1.79.....	7070.10	400	II	1p <sup>3</sup> —1s <sup>3</sup>	
	6878.35	300	II		
	6791.05	100	II		
	4962.24	40	I	1p <sup>3</sup> —2d <sup>3</sup>	
	4872.49	40	I		
	4832.08	50r	I		*
	4811.87	40r	I	1p <sup>3</sup> —1p' <sup>3</sup>	
	4784.32	30	I		
	4741.92	30	I		*
	4722.27	30	I		
2.27.....	6617.27	50	III	1d <sup>3</sup> —1f'' <sup>3</sup>	
	6503.99	80	II		
	6408.47	100	II		*

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Strontium—Continued					
2.27—Cont.	5521.75	25	II	1d <sup>3</sup> —1d <sup>3</sup> *	
	5504.17	30	II		
	5480.84	40	II		
	5256.90	50	II	1d <sup>3</sup> —1p <sup>3</sup> *	
	5238.54	30	II		
	5225.11	20	II		
Ionized Strontium, Sr <sup>+</sup> , I.P. 10.98					
0.0.....	4215.52	300r	II	1s <sup>2</sup> —1p <sup>2</sup>	
	4077.71	400r	II		
1.82.....	10915.4	.....	.....	1d <sup>2</sup> —1p <sup>2</sup> *	
	10328.0	.....			
3.03.....	4305.46	40	III	1p <sup>2</sup> —2s <sup>2</sup>	
	4161.81	30	IV?		
	3474.90	10	V	1p <sup>2</sup> —2d <sup>2</sup>	
	3464.47	50	V		
	3380.72	50	V		
	Barium, Ba, I.P. 5.19 (6)				
0.0.....	7911.36	3	I A	(1s—1p <sup>3</sup> )	
	5535.53	1000R	I	1s—1p	
	3501.11	200R	II	1s—1p''	
1.18.....	7672.12	25r	I	1d <sup>3</sup> —1f <sup>3</sup> *	
	7280.31	150r	II		
	7060.00	400r	II		
	6595.35	200	I	1d <sup>3</sup> —1d <sup>3</sup> *	
	6527.32	250	I		
	6498.78	300r	II		
	6110.81	300r	II	1d <sup>3</sup> —1p <sup>3</sup> *	
	6063.15	200	II		
	6019.50	100	II		
	3993.40	80	II	1d <sup>3</sup> —3f <sup>3</sup> *	
	3935.72	50	II		
	3909.92	40	II		
	3579.67	80n	III	1d <sup>3</sup> —4f <sup>3</sup> *	
	3544.66	80n	III		
	3524.97	80n	III		
1.41.....	5826.29	150	II	1D—1P''	
1.56.....	7905.80	6	III	1p <sup>3</sup> —1s <sup>3</sup>	
	7392.44	30	II		
	7195.26	80	III		
	5777.70	400r	II	1p <sup>3</sup> —2d <sup>3</sup> *	
	5519.12	200	II		
	5424.62	100	II		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Barium, $Ba^+$ , I.P. 9.96					
0.0.....	4934.10 4554.04	700R 1000R	II II	$1s^2-1p^2$	
0.65.....	6496.90 6141.76 5853.70	600r 600r 200	III III III	$1d^2-1p^2$	
2.71.....	4899.97 4524.95 4166.02 4130.68 3891.79	35 35 20 80 50	IV? V V V V	$1p^2-2s^2$  $1p^2-2d^2$	
Zinc, $Zn$ , I.P. 9.35					
0.0.....	3075.88 2138.61	(8R) (8R)	.....	$(1s-1p^3)$ $1s-1p$	
4.01.....	4810.53 4722.16 4680.20 3344.91 3302.56 3282.28	(10R) (10R) (10R) (10R) (8R) (8R)	.....	$1p^3-1s^3$  $1p^3-2d^3$ *	
5.77.....	6362.37 4029.88	(10) (8r)	.....	$1p-2D$ $1p-3D$	
Ionized Zinc, $Zn^+$ , I.P. 18.2					
0.0.....	2061.96 2025.49	(7) (8)	.....	$1s^2-2p^2$	
6.10.....	6214.65 5894.43	(5) (6)	.....	$2p^2-md^2?$	
Cadmium, $Cd$ , I.P. 8.95					
0.0.....	3261.04 2288.02	(10) (10R)	.....	$(1s-1p^3)$ $1s-1p$	
3.78.....	5085.88 4799.91 4678.19 3610.51 3466.18 3403.60	(10R) (10R) (10R) (10R) (10R) (10R)	.....	$1p^3-1s^3$  $1p^3-2d^3$ *	
5.39.....	6438.47	(10)	.....	$1p-2D$	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Cadmium, $Cd^+$ , I.P. 17.3					
0.0.....	2265.04 2144.39	(8R) (8R)	.....	$1s^2-1p^2$	
Mercury, $Hg$ , I.P. 10.39					
0.0.....	2536.52 1849.57	(10n) .....	.....	$(1s-1p^3)$ $1s-1p$	
5.44.....	5460.74	(10)	.....	$1p^3-1s^3$	
4.87.....	4358.34	(10)	.....		
4.65.....	4046.56	(10)	.....		
6.67.....	5790.66 5769.60	(10r) (10r)	.....	$1p-2d$ $(1p-2d^3)$	
Boron, $B$ , I.P. 7:					(5)
0.0.....	2497.7 2496.8	(10) (10)	.....	$1p^2-1s^2$	
Aluminium, $Al$ , I.P. 5.96					
0.0.....	3961.54 3944.03 3092.84 3092.72 3082.16	(10R) (10R) (6R) (10R) (10R)	.....	$1p^2-1s^2$ $1p^2-2d^2$	
3.13.....	6698.73 6696.06	(3) (3)	.....	$1s^2-3p^2$	
Ionized Aluminium, $Al^+$ , I.P. 18.18					(7)
0.0.....	3057.16 1670.98	(10) (20)	.....	$(1s-1p^3)$ $1s-1p$	
4.04.....	1862.48 1858.13 1856.00	(10) (7) (3)	.....	$1p^3-2s^3$	
10.69.....	7063.62 7056.56 7042.06	(3) (4) (5)	.....	$2s^3-2p^3$	
11.09.....	3587.44 3587.06 3586.55	(7) (8) (9)	.....	$1d^3-2f^3$ *	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Doubly Ionized Aluminium, $Al^{++}$ , I.P. 28.32 (8)					
0.0.....	1854.67 1862.90	(10) (10)	.....	$1s^2-1p^2$	
6.64.....	1611.94 1605.72	(8) (8)	.....	$1p^2-1d^2$	
14.32.....	3612.35 3601.62	(15) (20)	.....	$1d^2-1p^2$	
Gallium, $Ga$ , I.P. 5.97					
0.0.....	4172.06 4033.03 2944.18 2943.66 2874.24	(30R) (30R) (5R) (10R) (10R)	.....	$1p^2-1s^2$  $1p^2-2d^2$	
Indium, $In$ , I.P. 5.76					
0.0.....	4511.27 4101.72 3258.52 3256.03 3039.34	(10R) (8R) (6R) (10R) (10R)	.....	$1p^2-1s^2$  $1p^2-2d^2$	
Thallium, $Tl$ , I.P. 6.08					
0.0.....	3775.72 2767.67	(10R) (10R)	.....	$1p^2_1-1s^2$ $1p^2_1-1d^2$	
0.96.....	5350.46 3529.43 3519.24	(10R) (8R) (10R)	.....	$1p^2_1-1s^2$ $1p^2_1-2d^2$	
Ionized Carbon, $C^+$ , I.P. 24.28 (9)					
0.0.....	1036.84 1036.22 858.2	(5) (5) (7)	.....	$1p^2-x$ $1p^2-2s^2$	
11.90.....	2837.60 2836.71	(8) (10)	.....	$x-2p^2$	
14.37.....	6582.85 6578.03	(8) (10)	.....	$2s^2-2p^2$	
16.24.....	7236.19 7231.12 3920.77 3919.06	(8n) (6n) (8) (6)	.....	$2p^2-2d^2$ $2p^2-3s^2$	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Carbon—Continued					
17.94.....	4267.27	(10n)	.....	2d <sup>2</sup> —3f <sup>2</sup>	
	4267.02	(8n)	.....		
	5891.65	(2)	.....	2d <sup>2</sup> —3p <sup>2</sup>	
	5889.97	(3)	.....		
Treble Ionized Silicon, Si <sup>+++</sup> , I.P. 44.95 (10)					
0.0.....	1393.9	(10)	.....	1s <sup>2</sup> —1p <sup>2</sup>	
	1402.9	(8)	.....		
23.98.....	4088.86	(10)	.....	2s <sup>2</sup> —2p <sup>2</sup>	
	4116.10	(8)	.....		
Lead, Pb, I.P. 7.38 (11)					
0.0.....	2833.07	(6R)	.....	2p <sub>4</sub> —2s	
	2170.00	(6R)	.....	2p <sub>4</sub> —3d	
0.96.....	3639.58	(6R)	.....	2p <sub>3</sub> —2s	
	2657.10	(2)	.....	2p <sub>3</sub> —3d	
1.32.....	4057.83	(5R)	.....	2p <sub>3</sub> —2s	
	2873.32	(4R)	.....	2p <sub>3</sub> —3d	
2.64.....	7228.97	(4)	.....	2p <sub>1</sub> —2s	
	4168.04	(3)	.....		
	4062.15	(3)	.....	2p <sub>1</sub> —3d	
	4019.64	(3)	.....		
	3671.50	(3R)	.....	2p <sub>1</sub> —3s	
	3572.74	(5R)	.....	2p <sub>1</sub> —x	
3.64.....	5201.47	(3)	.....	y—3s	
	5005.44	(4)	.....	y—x	
Oxygen, O, I.P. 13.56 (12)					
0.0.....	1355.67	.....	.....	(op <sup>3</sup> —1s <sup>5</sup> )	
	1358.60				
	1306.09	.....	.....	op <sup>3</sup> —1s <sup>3</sup>	
	1304.95				
	1302.25				
9.11.....	7775.68	(6)	.....		
	7774.01	(8)	.....	1s <sup>5</sup> —1p <sup>5</sup>	
	7771.97	(10)	.....		
	3947.61	(4)	.....		
	3947.51	(7)	.....	1s <sup>5</sup> —2p <sup>5</sup>	
	3947.33	(10)	.....		
9.48.....	8446.38	(10)	.....	1s <sup>3</sup> —1p <sup>3</sup>	
	4638.30	(10)	.....	1s <sup>3</sup> —2p <sup>3</sup>	



TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Oxygen—Continued					
10.69.....	6456.07	(9)	.....	$1p^5-3s^5$	
	6454.55	(7)			
	6453.69	(6)			
	6158.20	(10)	.....	$1p^5-3d^5$	
	6156.78	(8)			
	6155.99	(7)			
10.94.....	7254.05	(2)	.....	$1p^3-3s^3$	
	7002.22	(4)	.....	$1p^3-3d^3$	
	6046.34	(7d)	.....	$1p^3-4s^3$	
	5958.53	(6d)	.....	$1p^3-4d^3$	
Sulphur, S, I.P. 10.31 (13)					
0.0.....	1914.96	.....	.....	$(op^3-1s^5)$	
	1900.47				
	1826.35				
	1820.53	.....	.....	$op^3-1s^3$	
	1807.42				
6.50.....	9237.71	.....	.....	$1s^5-1p^5$	
	9228.17				
	9212.80				
	4696.31	(6)	.....	$1s^5-2p^5$	
	4695.51	(8)			
	4694.18	(10)			
7.84.....	6757.16	(7)	.....	$1p^5-4d^5$	
	6748.83	(6)			
	6743.69	(5)			
Selenium, Se, I.P. ?					
?	4742.58	(8)	.....	$1s^5-2p^5$	
	4739.28	(9)			
	4731.04	(10)			
Scandium, Sc, I.P. 5.4: (14)					
0.0.....	6448.10	I	II A	$1d^2-1d^{1/2}$	(28)
	6413.37	50	I A		(15)
	6378.83	40	I A		
	6344.84	5	II A	$1d^2-2d^{1/2}$	
	6305.99	20	I A		
	6305.69	400	I A		
	6239.78	100	I A	$1d^2-3d^{1/2}$	
	6239.40	20	II A		
	6276.30	15	II A		
	6258.97	100	I A		
	6210.68	200	I A		
	6193.66	(3)	II A		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Scandium—Continued					
o.o.—Cont.	5349.71	15	II A	$1d^2-1p'^2$	
	5342.96	10	II A		
	5301.94	2	II A		
	4791.51	4	I A	$1d^2-1f'^2$	
	4779.35	20	I A		
	4753.16	15	I A		
	4082.40	40	I	$1d^2-4d'^2$	
	4054.54	35	I		
	4047.79	25	I	$1d^2-5d'^2$	
	4023.69	100	II		
	4020.40	75	II		
	3996.61	30	I	$1d^2-2f'^2$	
	3933.37	20	II		
	3911.81	100	II		
	3907.49	75	II	$1d^2-2p'^2$	
	3273.64	20	II A		
	3269.91	15	II A		
	3255.69	6	I A		
?.....	5724.00	15	III A	$1f^4-1g'^4$ *	
	5717.30	15	III A		
	5711.75	100	III A		
	5708.62	15	III A		
	5700.14	100	II	$1f^4-1f'^4$ *	
	5686.84	150	II		
	5671.80	200	II		
	5086.95	40	II		
	5085.54	40	II	$1f^4-1d'^4$ *	
	5083.72	80	II		
	5081.56	125	II		
	4743.81	40	III		
	4741.02	30	III	$1f^4-1d'^4$ *	
	4737.65	20	III		
	4734.10	15	III		
	4729.23	30	III		
	4728.76	5	III		
2. 10.....	6198.44	(2)	IV A	$1f^2-3f'^2$	
	6146.25	(3)	IV A		
	6026.18	15	IV	$1f^2-6d'^2$ *	
	5988.42	20	IV		
Ionized Scandium, $Sc^+$ , I.P.?					
o.o.....	3666.54	2	III	$1d^3-1f'^3$	
	3651.80	7	III		
	3645.31	10	III		
	3642.79	25	II		
	3630.76	12	II		
	3613.84	30	II		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes		
Ionized Scandium—Continued							
0.0—Cont..	3590.48	6	II	Id <sup>3</sup> —Id' <sup>3</sup>			
	3580.64	6	II				
	3580.94	7	II				
	3576.35	8	II				
	3572.53	20	II				
	3567.70	6	II				
	3558.55	7	II				
	3372.14	12	III	Id <sup>3</sup> —Ip' <sup>3</sup>			
	3368.94	8	III				
	3361.94	6	III				
	3361.26	6	III				
	3359.68	6	III				
	3352.05	2	II				
?.....	4246.83	75	III	?	(16)		
0.60.....	4431.35	4	V	If <sup>3</sup> —If' <sup>3</sup>			
	4420.66	(1)	...				
	4415.55	40	III				
	4400.38	50	III				
	4384.80	8	IV				
	4374.46	60	III	If <sup>3</sup> —Id' <sup>3</sup> *			
	4354.60	8	V				
	4325.00	50	III				
	4320.73	75	III				
	4314.09	100	III				
	4305.71	10	IV				
	4294.77	10	IV				
	1.50.....	6320.86	(2)	...		Ip <sup>3</sup> —Id' <sup>3</sup> *	
6309.90		(2)	...				
6300.70		(1)	...				
6279.74		(3)	...				
6245.64		(4)	V	Ip <sup>3</sup> —Ip' <sup>3</sup>			
5684.21		8	V				
5669.05		2	IV				
5667.16		2	V				
5658.35		2	V				
5657.89		15	V				
5640.99		2	V				
Yttrium, Y, I.P. 5?							
0.0.....		6435.02	(5)	.....	Id <sup>2</sup> —md' <sup>2</sup>		
	6402.02	(3)					
	6222.58	(4)					
	6191.71	(1)	.....	Id <sup>2</sup> —mp' <sup>2</sup>			
	6138.43	(4)					
	6023.41	(3)					
	5944.86	(1)					

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Yttrium—Continued					
o. o—Cont.	4760.99	(5)	.....	$1d^2-mf'^2$	(29)
	4674.84	(10)			
	4643.70	(10)			
	4174.14	(8)	.....	$1d^2-np'^2$	
	4142.87	(20)			
	4083.71	(8)			
	4128.32	(30)	.....	$1d^2-n'p'^2$	
	4102.38	(20)			
	4039.83	(5)			
	Ionized Yttrium, $Y^+$ , I.P. ?				
o. o.....	4422.60	(10)	.....	$1d^3-1p'^3$	
	4398.03	(15)			
	4358.72	(8)			
	4309.61	(20)			
	4235.71	(6)			
	4199.28	(3)	.....	$1d^3-1f'^3$	
	3788.69	(30)			
	3774.33	(50)			
	3710.30	(50)			
	3611.05	(30)			
o. 93.....	3601.91	(20)	.....	$1d^3-1d'^3$	
	3600.72	(50)			
	4900.11	(30)	.....	$1f^3-1d'^3$	
	4883.69	(50)			
	4854.88	(30)			
	4823.32	(10)			
	4786.57	(5)	.....	$1f^3-1f'^3$	
	5320.79	(1)			
	5289.82	(2)			
	5205.72	(10)			
5200.41	(10)				
5119.10	(3)	.....			
5087.43	(10)				
4982.12	(3)				
Titanium, $Ti$ , I.P. 6.5 (17)					
o. o.....	6413.30	.....	II A	$(1f^3-1g'^3)$	
	6359.90	o	II A		
	6325.17	1	II A		
	6296.68	2	II A		
	6295.27	1	II A		
	6273.42	.....	II A		
	6257.80	.....	II A		
	6031.71	o	II A		
	5984.59	.....	II A		
	5944.65	1	II A		
	5940.66	1	II A		
	5913.71	o	II A		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Titanium—Continued					
o.o—Cont.	5490.82	1	I A	(1f <sup>3</sup> —1d' <sup>5</sup> )	
	5460.51	4	I A		
	5446.67	2	II A		
	5426.27	3	I A		
	5408.93	(1)	I A		
	5396.57	1	I A		
	5376.58	(0)	I A	1f <sup>3</sup> —1f' <sup>3</sup>	
	5252.11	8	I A		
	5219.72	8	I A		
	5210.39	40	I		
	5192.98	35	I		
	5173.74	30	I		
	5152.18	10	I A		
	5147.48	10	I A		
	5064.66	25	I		
	5039.96	22	I		
	5014.19	(25)	I	1f <sup>3</sup> —1d' <sup>3</sup> *	
	5009.65	7	I A		
	4997.10	8	I A		
	4715.31	4	II A	1f <sup>3</sup> —1g' <sup>3</sup>	
	4693.68	5	II A		
	4681.91	30	I		
	4667.59	25	I		
	4656.47	25	I		
	4562.63	6	II A	(1f <sup>3</sup> —1D')	
	4527.46	(4)	III A		
	4496.24	2	III A	(1f <sup>3</sup> —1F')	
	4462.08	2	III A		
	4112.73	20	II	(1f <sup>3</sup> —1G')	
	4076.38	4	III A		
	4024.57	35	II	1f <sup>3</sup> —2f' <sup>3</sup>	
	4008.94	35	II		
	3998.65	100R	II		
	3989.77	80R	II		
	3981.77	70R	II		
	3964.27	35	II		
	3962.85	35	II	1f <sup>3</sup> —2d' <sup>3</sup>	
	3958.21	80	II		
	3956.28	60	II		
	3948.66	60	II		
	3929.87	40	II		
	3924.51	50	II		
	3898.49	8	III		
	3771.64	25	I		
	3753.63	25	I		
	3752.87	80R	I		
	3741.06	60R	I	1f <sup>3</sup> —4f' <sup>3</sup>	
	3729.77	50R	I		
	3722.58	15	II		
	3717.39	20	I		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Titanium—Continued					
o. o—Cont.	3680.89	15	I	If <sup>3</sup> —3d' <sup>3</sup>	
	3668.95	15	I		
	3660.62	12	I		
	3654.58	15	I		
	3646.19	12	I		
	3637.97	10	II	If <sup>3</sup> —2g' <sup>3</sup>	
	3671.66	20	I		
	3658.14	20	I		
	3653.40	100R	I		
	3642.68	80R	I		
	3635.47	80R	I		
	3385.93	40R	II		
	3377.57	30R	I		
	3370.42	40R	II		
	3371.44	80R	II		
	3354.64	60R	II		
	3341.87	50R	II		
	3199.91	100R	II	If <sup>3</sup> —4g' <sup>3</sup>	*
	3192.00	80R	II		
	3186.45	60R	II		
o. 82.....	8435.64	(5)	.....	If <sup>3</sup> —1d' <sup>3</sup>	*
	8434.89	(4)			
	8426.46	(4)			
	8412.34	(3)			
	8396.85	(2)			
	5045.43	5	III A	If <sup>3</sup> —2g' <sup>3</sup>	
	5043.59	7	III A		
	5040.63	6	III A		
	5024.85	20	II		
	5022.87	25	II		
	5020.04	25	II		
	5016.17	20	II		
	5014.28	(25)	I		
	5007.22	40	II		
	4999.51	45	II		
	4991.08	50	II		
	4981.75	60	II		
	4555.50	30	II		
	4552.56	35	II		
	4548.77	35	II		
	4544.70	30	II		
	4536.00	40	II	If <sup>3</sup> —2f' <sup>3</sup>	
	4535.92	40	II		
	4535.58	50	II		
	4534.78	60	II		
	4533.25	80	II		
	4527.32	35	II		
	4522.80	40	II		
	4518.03	50	II		
	4512.74	40	II		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Titanium—Continued					
0.82—Cont.	4305.91	60	II	1f <sup>3</sup> —2d' <sup>3</sup>	
	4301.08	50	II		
	4300.55	50	II		
	4298.67	40	II		
	4295.75	22	II		
	4290.93	22	II		
	4289.08	25	II		
	4287.41	22	II		
	4286.01	25	II		
	4281.40	10	III		
	4274.60	15	III		
	4272.44	8	III A		
	4326.35	9	II	(1f <sup>3</sup> —5d' <sup>3</sup> )	
	4314.80	25	II		
	4299.64	15	III		
	4288.18	3	III A		
0.90.....	6743.14	10	III A	1D—1D'	
	6599.12	12	III A	1D—1F'	
	4958.25	2	III A	(1D—3d' <sup>3</sup> )	
	4840.88	25	I	1D—2D'	
	3904.77	40n	II	1D—2F'	
	3786.03	20	II	1D—1P'	
1.05.....	8692.29	(1)	.....	1p <sup>3</sup> —1d' <sup>3</sup>	
	8682.93	(2)			
	8675.33	(2)			
	6126.21	20	II	1p <sup>3</sup> —1s' <sup>3</sup>	
	6085.22	20	II		
	6064.64	9	II A		
	5941.73	12	II A	1p <sup>3</sup> —2d' <sup>3</sup>	
	5937.79	6	III A		
	5922.11	18	II		
	5899.29	25	II		
	5866.45	35	II		
	5295.78	4	III	1p <sup>3</sup> —3d' <sup>3</sup>	
	5284.39	2	III A		
	5282.38	3	III A		
	4723.18	10	III	1p <sup>3</sup> —5d' <sup>3</sup>	
	4722.63	10	III		
	4710.19	18	III		
	4698.79	20	II		
	4691.34	20	II		
	4082.46	20	III	1p <sup>3</sup> —3p' <sup>3</sup>	
	4078.47	30	III		
	4065.11	15	III		
	4064.22	15	III		
	4060.27	20	III		
	4055.02	20	III		



TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Titanium—Continued					
I. 05—Cont.	3725.12	20	III	1p <sup>3</sup> —2s' <sup>3</sup>	
	3709.95	20	III		
	3702.29	10	III		
I. 44.....	7364.11	2	IV	2f <sup>3</sup> —2f' <sup>3</sup>	
	7357.73	3	IV		
	7344.69	4	IV		
	7299.67	(2)	...		*
	7271.47	(0)	...	2f <sup>3</sup> —2d' <sup>3</sup>	
	7251.74	8	III		
	7244.82	10	III		
	7209.44	20	III		
	7188.59	(1)	...	2f <sup>3</sup> —4f' <sup>3</sup>	
	7138.92	(1)	...		
	6556.08	25	III		
	6554.23	20	III		
	6546.26	20	III	2f <sup>3</sup> —3d' <sup>3</sup>	
	6508.15	3	III A		
	6497.71	3	III A		
	6366.38	8	III		
	6336.10	8	III	2f <sup>3</sup> —2g' <sup>3</sup>	
	6318.00	5	III A		*
	6312.23	10	III		
	6303.76	10	III		
	6261.10	35	II	2f <sup>3</sup> —5d' <sup>3</sup>	
	6258.70	50	II		
	6258.10	40	II		
	5514.54	25	II		
	5514.36	20	II	2f <sup>3</sup> —3g' <sup>3</sup>	
	5512.54	25	II		
	5481.90	5	I A		
	5471.19	5	III		
	5474.28	6	II	2f <sup>3</sup> —6d' <sup>3</sup>	
	5453.67	3	III A		
	5438.33	1	III A		
	5145.47	12	II		
	5113.45	10	III	2f <sup>3</sup> —4g' <sup>3</sup>	
	5087.07	8	III		*
	5038.41	25	II		
	5036.47	25	II		
	5035.92	25	II	2f <sup>3</sup> —6f' <sup>3</sup>	
	4457.42	40	II		
	4455.32	30	II		
	4453.32	30	II		*
I. 50.....	7949.11	(3)	.....	1G—1G'	
	4820.42	20	II	1G—2F'	
	4427.10	40	III	1G—2G'	
	4186.12	25	III	1G—3G'	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Titanium—Continued					
I. 73.....	4656.06	6	III	1p <sup>5</sup> —3d' <sup>5</sup>	
	4650.02	10	III		
	4645.19	12	III		
	4639.94	15	III		
	4639.66	15	III		
	4639.36	18	III		
	4629.34	15	III		
	4623.10	25	III		
	4617.27	30	II		
	4496.15	20	III	1p <sup>5</sup> —1p' <sup>5</sup>	
	4480.10	20	III		
	4481.27	30	III		
	4480.61	5	III A		
	4479.70	9	III		
	4471.24	20	III		
	4465.80	20	III		
I. 87.....	8548.06	(2)	.....	1g <sup>3</sup> —4f' <sup>3</sup>	
	8518.20	(4)			
	8468.45	(3)			
	8068.21	(2)	.....	1g <sup>3</sup> —2g' <sup>3</sup>	
	8024.83	(2)			
	7978.87	(4)			
	6146.22	3	III	1g <sup>3</sup> —4g' <sup>3</sup>	
	6121.03	3	III		
	6092.79	4	III		
	5996.01	2	III	1g <sup>3</sup> —1h' <sup>3</sup>	
	5988.58	2	III		
	5978.54	25	II		
	5965.83	30	II		
	5953.16	30	II	1g <sup>3</sup> —6f' <sup>3</sup> *	
	5297.29	6	III		
	5283.45	8	III		
	5265.97	10	III		
	4925.42	5	IV	1g <sup>3</sup> —2h' <sup>3</sup>	
	4915.24	5	III		
	4913.63	20	III		
	4899.93	20	III		
	4885.09	20	II		
	4453.70	20	III	1g <sup>3</sup> —5g' <sup>3</sup> *	
	4450.90	25	III		
	4449.13	30	III		
I. 98.....	5025.58	18	III	1g' <sup>5</sup> —2f <sup>5</sup> *	
	5013.30	18	III		
	5001.01	10	III A		
	4989.16	10	III		
	4978.21	10	III		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Titanium—Continued					
2.08.....	5224.96	8	III	1f' <sup>5</sup> —2f <sup>5</sup> *	
	5224.57	6	III		
	5224.32	15	III		
	5223.64	6	III		
	5222.68	6	III		
2.16.....	6186.15	3	III	1d <sup>3</sup> —5f' <sup>3</sup> *	
	6149.81	2	III		
	6138.38	1	IV		
	4948.21	3	IV	1d <sup>3</sup> —7f' <sup>3</sup>	
	4941.58	3	IV		
	4928.36	12	III		
	4921.79	12	III		
	4919.88	10	III		
2.24.....	5999.66	8	III	(1h <sup>3</sup> —2G')	
	5565.48	9	III	(1h <sup>3</sup> —3G')	
	4778.27	10	III	(1h <sup>3</sup> —4G')	
	7496.12	(2)	.....	1h <sup>3</sup> —4g' <sup>3</sup>	
	7489.61	(2)			
	7440.58	(3)			
	5740.04	4	IV	1h <sup>3</sup> —2h' <sup>3</sup>	
	5739.50	9	III		
	5715.13	9	III		
	4893.06	2	IV	1h <sup>3</sup> —1i' <sup>3</sup>	
	4882.34	2	IV		
	4870.14	20	III		
	4868.28	18	III		
	4856.01	20	III		
	4769.78	4	III	1h <sup>3</sup> —3h' <sup>3</sup>	
	4766.33	4	III		
	4759.28	25	III		
	4758.13	25	III		
	4747.69	3	III		
	4742.80	20	III		
	4734.68	3	IV		
2.26.....	6861.47	6	III		
	6091.17	20	III	2G—2G'	
	5644.14	18	III	2G—3G'	
	4799.81	12	III	(2G—3h' <sup>3</sup> )	
2.28.....	5720.49	3	III	1d' <sup>5</sup> —2f <sup>5</sup>	
	5716.48	4	III		
	5713.93	3	IV		
	5711.94	4	IV		
	5708.27	3	IV		
	5702.69	6	III		
	5689.47	10	III		
	5675.41	9	III A		
	5662.16	12	III		

[illegible]

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes		
Ionized Titanium—Continued							
O. 13.....	3500.47	4	IV	2f <sup>4</sup> —1g' <sup>4</sup>			
	3491.05	8ru	III				
	3480.74	4	IV			2f <sup>4</sup> —1f' <sup>4</sup>	
	3477.18	15ru	III				
	3476.99	2	V				
	3461.50	20ru	III				
	3444.31	15ru	III				
	3348.82	5	III				
	3346.72	7	III				
	3343.76	6	III				
	3340.33	15ru	III				
	3335.19	20ru	III				
	3329.45	20ru	III				
	3326.76	5ru	III				
	3322.93	20ru	III				
	3318.01	8ru	III				
	3308.79	8ru	III				
	3168.52	30ru	III				
	3162.56	25ru	III				
	3161.76	20ru	III				
	3161.19	20ru	III				
	3155.65	10ru	IV				
	3154.18	10ru	IV				
	3152.24	12ru	IV				
O. 59.....	4025.13	4	V	(1f <sup>2</sup> —1g' <sup>4</sup> ) *			
	4012.40	10	V				
	3761.33	40r	IV	1f <sup>2</sup> —1f' <sup>2</sup>			
	3759.30	40r	IV				
	3721.64	8	V	1f <sup>2</sup> —1d' <sup>2</sup>			
	3685.19	40r	IV				
	3372.20	5	V	1f <sup>2</sup> —1g' <sup>2</sup>			
	3349.02	20r	III				
	3341.87	50r	II				
	I. 08.....	4450.49	4			V	1d <sup>2</sup> —1f' <sup>2</sup>
		4443.80	25	V			
		4395.04	25	V			
4344.31		3	V	1d <sup>2</sup> —1d' <sup>2</sup>			
4337.92		10	V				
4294.10		8	V				
4287.88		2	V				
I. 12.....	4501.27	25	V	1g <sup>2</sup> —1f' <sup>2</sup>			
	4468.49	25	V	(1g <sup>2</sup> —1d' <sup>2</sup> )			
	4341.39	2	V				
	3913.45	40	V	1g <sup>2</sup> —1g' <sup>2</sup>			
	3900.53	50	V	* 1g <sup>2</sup> —1g' <sup>2</sup>			
	3232.26	8ru	IV	1g <sup>2</sup> —2f' <sup>2</sup>			
	3229.40	10ru	V				

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Titanium—Continued					
I. 16.....	4417.71	8	V	( $1p^4 - 1d^2$ )	
	4330.71	1	V	$1p^4 - 1d'^4$	
	4320.95	1	V		
	4314.08	5	V		
	4312.88	7	V		
	4307.89	12	V		
	4301.93	5	V		
	4300.05	12	V		
	4290.23	8	V		
I. 22.....	4708.65	(2)	.....	( $1p^3 - 1f'^2$ )	
	4589.96	3	V	$1p^2 - 1d'^2$	
	4563.77	15	V		
	4533.97	20	V		
	4399.77	6	V	( $1p^2 - 1d'^4$ )	
	4394.06	2	V	$1p^2 - 1s'^2$	
	3641.33	10	V		
	3624.84	8	V		
I. 56.....	5418.77	1	V	$2d^2 - 1f'^2$	
	5381.02	(4)	.....		
	5336.78	2	V		
	5262.14	(1)	.....	$2d^2 - 1d'^2$	
	5226.56	3	V		
	5188.70	4	V		
	5154.07	(1)	.....	$2d^2 - 2d'^2$	
	3757.68	8	V		
	3741.63	8ru	V		
I. 57.....	4571.98	15	V	$1h^2 - 1g'^2$	
	4549.64	25	V		
	4529.46	1	V		
	3224.23	8ru	V	$1h^2 - 2g'^2$	
	3218.25	8ru	IV		
I. 88.....	5185.90	(5)	.....	$2g^2 - 1g'^2$	
	5129.17	(8)	.....		
	4053.84	4	V	$2g^2 - 2f'^2$	
	4028.35	5	V		
	3510.84	10r	V	$2g^2 - 2g'^2$	
	3504.89	8r	V		
2. 05.....	6559.58	(1)	.....	$2p^2 - 1d'^2$	
	6491.68	(2)	.....		
	4805.11	4	V	$2p^2 - 1s'^2$	
	4779.99	2	V		
2. 58.....	4386.84	(5)	.....	$2f^2 - 2g'^2$	
	4367.67	1	V		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes		
Ionized Titanium—Continued							
3. 10. ....	5072. 30	(3)	.....	$3d^2-3d'^2$			
	5069. 12	(1)					
	5010. 21	(1)					
	4911. 19	(1)	.....	$3d^2-2p'^2$			
	4873. 95	(3)					
Zirconium, Zr, I.P. 6? (27)							
0. 0. ....	6143. 23	(6)	.....	$1f^3-md'^3$			
	6134. 58	(5)					
	6127. 49	(6)					
	5935. 24	(3)					
	5885. 61	(2)					
✓	3891. 39	(6)	.....	$1f^3-mf''^3$ *			
	3863. 57	(7)					
	3835. 96	(4)					
	?	7169. 15	(8)	.....		$1f^3-md'^3$ *	
		7103. 77	(6)				
7102. 96		(6)					
7097. 78		(7)					
7087. 36		(6)	.....	$1f^3-mg'^3$ *			
4851. 36		(6)					
4824. 30		(7)					
4815. 62		(7)					
4805. 89		(7)	.....	$1f^3-nd'^3$ *			
4788. 70		(7)					
4772. 33		(8)					
4739. 49		(8)					
4710. 08		(8)	.....	$1p^3-ms'^3$			
4687. 81		(8)					
?	5797. 77	(7)	.....	$1p^3-nd'^3$ *			
	5680. 94	(7)					
	5623. 57	(5)					
	4539. 97	(7)	.....	$1p^3-nd'^3$ *			
	4535. 75	(6)					
4553. 02	(6)	Ionized Zirconium, Zr <sup>+</sup> , I.P.?					
0. 0. ....	3572. 46	(8)	.....	$1f^4-1g'^4$ *			
	3496. 17	(9)					
	3438. 23	(9)					
	3391. 96	(8)					
	3357. 27	(7)	.....	$1f^4-1f'^4$ *			
	3306. 28	(7)					
	3279. 26	(7)					
	3273. 05	(7)					
	3165. 98	(7)	.....	$1f^4-1d'^4$ *			
	3138. 66	(7)					
	3129. 70	(6)					
	3125. 92	(6)					



TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes			
Ionized Zirconium—Continued								
0.40.....	3934.14	(6)	.....	$2f^4-1g'^4$ *				
	3843.01	(5)						
	3766.72	(5)						
	3697.46	(5)						
	3674.72	(8)	.....	$2f^4-1f'^4$ *				
	3614.78	(7)						
	3576.85	(7)						
	3556.59	(7)						
	3430.54	(7)	.....	$2f^4-1d'^4$ *				
	3410.25	(7)						
	3404.84	(6)						
	3399.35	(6)						
Vanadium, V, I.P.?								
(19)								
0.0.....	5632.47	I	II A	$(1f^4-1d'^6)$ *				
	5592.96	I	II A					
	4881.55	50	I	$1f^4-1d'^4$				
	4875.46	40	I					
	4864.74	40	I					
	4851.48	40	I					
	4832.43	30	I					
	4831.64	35	I					
	4827.45	30	I					
	4799.78	5	II A					
	4784.48	5	II A	$1f^4-1g'^4$ *				
	4635.18	15	I					
	4619.77	25	I					
	4606.15	15	I					
	4594.10	60	I					
	4586.36	50	I					
	4580.39	40	I					
	4577.17	40	I					
	4368.05	10	I	$1f^4-1f'^4$				
	4355.95	10	I					
	4352.89	50	I					
	4341.02	40	I					
	4332.83	30	I					
	4330.03	30	I					
	4309.80	20	I					
	4307.18	12	I					
	4306.22	15	I	$1f^4-2f'^4$ *				
	3902.26	50r	I					
	3875.08	35	I					
	3864.86	35	I					
	3855.37	30	I	$1f^4-2d'^4$ *				
	3855.86	60r	I					
	3840.76	60r	II					
	3828.56	60r	II					
	3822.01	30	I					
	3818.24	60	II					
	3813.49	60	II					
	3808.52	40	II					

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Vanadium—Continued					
o. o—Cont..	3185.40	40r	II	If <sup>4</sup> —2g' <sup>4</sup>	
	3184.02	25r	II		
	3183.96	35r	II		
	3183.41	30r	II		
o. 28.....	6296.52	15	I		
	6292.86	20	I		
	6285.19	20	I		
	6274.67	15	I		
	6258.80	.....			
	6258.60	8	II A	Id <sup>6</sup> —Id' <sup>6</sup>	
	6256.91	8	II A		
	6251.83	30	I		
	6243.11	30	I		
	6242.80	15	I		
	6230.73	30	I		
	6216.37	30	I		
	6199.20	30	I		
	6268.84	8	II A		
	6266.32	7	II A		
	6261.24	5	II A		
	6245.21	2	II A		
	6240.14	6	II A	Id <sup>6</sup> —If' <sup>6</sup>	
	6233.19	12	I A		
	6224.50	15	I		
	6213.87	15	I		
	6189.35	3	II A		
	6170.34	8	I A		
	6150.13	15	I		
	4460.30	50	I		
	4459.77	30	I		
	4457.48	15	I		
	4444.22	20	I		
	4441.69	25	I	Id <sup>6</sup> —Ip' <sup>6</sup>	
	4437.84	20	I		
	4436.14	15	I		
	4428.52	15	I		
	4419.94	12	I		
	4429.80	15	I		
	4426.01	15	I		
	4421.58	20	I		
	4416.48	20	I		
	4408.51	45	I		
	4408.21	70	I		
	4407.65	70	I		
	4406.65	80	I	Id <sup>6</sup> —2f' <sup>6</sup>	
	4400.59	60	II		
	4395.24	80	II		
	4389.99	100	II		
	4384.73	125r	II		
	4379.24	150r	II		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Vanadium—Continued					
0.28—Cont.	4209.86	20	I	(1d <sup>6</sup> —2f' <sup>4</sup> ) *	
	4191.55	10	II		
	4189.85	12	I		
	4182.60	10	I		
	4134.50	60	I	1d <sup>6</sup> —2d' <sup>6</sup>	
	4132.02	60	I		
	4128.08	60	I		
	4123.56	60	I		
	4116.70	4	I A		
	4116.48	50	I		
	4115.18	60	I		
	4111.79	100R	I		
	4109.78	50	I		
	4105.17	60	I		
	4099.80	60	I		
	4092.69	50	I		
	3705.03	30	I	1d <sup>6</sup> —2p' <sup>6</sup>	
	3704.70	60	II		
	3703.57	100	II		
	3695.87	40	II		
	3692.22	50	II		
	3690.27	40	II		
	3688.07	50	II		
	3683.11	30	I		
	3675.70	20	I		
1.05.....	8198.85	(3)	.....	1d <sup>4</sup> —1d' <sup>4</sup> *	
	8186.71	(3)			
	8161.06	(4)			
	8116.78	(5)			
	6812.42	2	III A	1d <sup>4</sup> —1f' <sup>4</sup> *	
	6785.02	3	III A		
	6766.53	4	III A		
	6753.03	5	III A		
	6135.36	15	II	1d <sup>4</sup> —1p' <sup>4</sup>	
	6119.51	40	I		
	6111.62	25	II		
	6090.18	50	I		
	6081.42	25	I		
	6058.11	5	II A		
	6039.69	25	I	1d <sup>4</sup> —2f' <sup>4</sup>	
	6002.60	4	II A		
	5782.60	2	III A		
	5761.41	2	III A		
	5743.44	18	II		
	5737.04	25	II		
	5727.66	20	II		
	5727.02	60	I		
	5706.97	30	I		
	5703.56	40	I		
	5698.51	60	I		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Vanadium—Continued					
1.05—Cont.	5668.37	12	II	1d <sup>4</sup> —2d' <sup>4</sup>	
	5657.45	12	II		
	5646.11	10	II		
	5627.63	30	I		
	5626.01	8	II		
	5624.90	10	II		
	5624.61	20	I		
	5604.94	8	II		
	5592.41	12	I		
	5584.49	10	I		
	4670.48	25	III	1d <sup>4</sup> —2p' <sup>4</sup>	
	4646.40	15	III		
	4640.74	7	III		*
	4640.06	8	III		
	4104.78	15	III	1d <sup>4</sup> —3f' <sup>4</sup>	
	4102.16	20	II		
	4095.49	25	II		*
	4090.59	25	I		
1.20.....	6624.86	5	III A	1p <sup>4</sup> —1p' <sup>4</sup>	
	6605.98	5	III A		
	6565.88	2	III A		
	6543.51	3	III A		
	6531.44	20	II		
	6504.16	8	II A	1p <sup>4</sup> —2d' <sup>4</sup>	
	6452.35	8	II A		
	6017.90	1	III A		
	6008.65	1	III A		
	6002.27	2	II A		
	5984.60	1	III A		*
	5980.74	2	III A		
2.05.....	4807.54	25	III	1g' <sup>6</sup> —mf <sup>6</sup>	
	4796.93	20	III		
	4786.51	20	III		
	4776.36	15	III		*
	4766.63	12	III		
	4757.50	8	III		
Chromium, Cr, I.P. 6.72 (20)					
0.0.....	4289.72	350R	II	1s <sup>7</sup> —1p <sup>7</sup>	
	4274.80	400R	II		
	4254.34	500R	II		
	3732.05	12	I	(1s <sup>7</sup> —1p <sup>6</sup> )	
	3730.84	10	I		
	3605.34	140R	II	1s <sup>7</sup> —2p <sup>7</sup>	
	3593.50	160R	II		
	3578.70	200R	II		
	3379.36	(1)	.....	(1s <sup>7</sup> —1p <sup>6</sup> )	
	3351.96	(2)			

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Chromium—Continued					
0.94.....	6362.83	15	I A}	(1s <sup>5</sup> —1p <sup>7</sup> )	
	6330.10	25	I A}		
	5208.42	300r	II}	1s <sup>5</sup> —1p <sup>5</sup>	
	5206.05	200r	II}		
	5204.51	150r	II}		
	4964.93	6	II A}	(1s <sup>5</sup> —2p <sup>7</sup> )	
	4942.48	8	II A}		
	4580.05	20	I}	1s <sup>5</sup> —1p <sup>5</sup>	
	4545.96	20	I}		
	4496.85	25	I}		
0.98.....	6630.01	4	I A}	(1d <sup>5</sup> —1p <sup>7</sup> )	(21)
	6580.91	.....	.....		
	6572.90	2	I A}		
	6537.90	3	I A}		
	6501.21	2	I A}		
	6467.18	.....	.....	1d <sup>5</sup> —1p <sup>5</sup>	
	6452.58	.....	.....		
	5409.83	100	I}		
	5348.33	50	I}		
	5345.81	70	I}		
	5300.72	25	I}		
	5298.26	60	I}		
	5296.70	50	I}		
	5265.72	25	I}		
	5264.16	50	I}		
	5247.57	40	I}		
	5123.46	6	I}	(1d <sup>5</sup> —2p <sup>7</sup> )	
	5091.86	3	II A}		
	5068.28	2	II A}		
	5048.78	2	II A}		
	4652.20	30	I}	1d <sup>5</sup> —1f <sup>5</sup>	
	4651.31	20	I}		
	4646.17	40	I}		
	4626.18	20	I}		
	4616.11	25	I}		
	4613.36	15	I}		
	4600.73	20	I}		
	4591.44	20	I}		
	4565.53	12	I}		
	4412.28	6	I A}		
	4391.77	8	I}		
	4384.99	20	I}		
	4373.26	8	I}		
	4371.32	20	I}		
	4359.66	20	I}		
	4351.82	60	I}		
	4351.06	20	I}		
	4344.52	40	I}		
	4339.74	20	I}		
	4339.46	40	I}		
	4337.58	30	I}		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Chromium—Continued					
0.98—Cont.	3941.53	20	I	1d <sup>5</sup> —1d <sup>5</sup>	
	3928.68	25	I		
	3921.07	20	I		
	3919.18	35 <sup>r</sup>	II		
	3916.27	12	I		
	3908.77	25	II		
	3903.15	8	I		
	3902.91	12	II		
	3894.06	15	I		
	3886.78	15	I		
	3885.22	15	I		
	3883.34	15	I		
2.88.....	7462.34	(10)	.....	1p <sup>7</sup> —2s <sup>7</sup>	
	7400.22	(10)			
	7355.93	(10)			
	5329.80	5n	II	1p <sup>7</sup> —1d <sup>7</sup>	
	5329.15	20n	II		
	5328.34	50n	II		
	5297.96	15n	II		
	5297.33	20n	II		
	5276.07	20n	II		
	5275.66	15n	II		
	5275.17	20n	II		
	4514.50	8	III	1p <sup>7</sup> —3s <sup>7</sup>	
	4491.68	3n	III		
	4475.36	8n	III		
	4129.21	20n	III		
	4111.67	20n	III	1p <sup>7</sup> —2d <sup>7</sup>	
	4111.46				
	4110.87				
	4098.18				
	4097.96	20n	III		
	4097.65				
3.30.....	9021.69	(4)	.....	1p <sup>5</sup> —2s <sup>5</sup>	
	9017.10	(5)			
	9009.95	(6)			
3.42.....	6981.04	(3)	.....	2p <sup>7</sup> —1d <sup>7</sup>	
	6980.81	(2)			
	6979.79	(10)			
	6925.99	(1)			
	6925.20	(9)			
	6924.14	(10)			
	6883.03	(9)			
	6882.38	(9)			
	6881.62	(9)			

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Molybdenum, <i>Mo</i> , I.P. 7.35 (22)					
0.0.....	3902.97 3864.12 3798.26 3466.83 3456.39	(50R) (50R) (50R) (8) (10)	.....	1s <sup>7</sup> —1p <sup>7</sup>   (1s <sup>7</sup> —1p <sup>8</sup> )	
1.32.....	5570.46 5533.01 5506.51	(25R) (30R) (40R)	.....	1s <sup>5</sup> —1p <sup>5</sup>	
Manganese, <i>Mn</i> , I.P. 7.41 (23)					
0.0.....	5432.55 5394.68 4034.49 4033.07 4030.76 2801.06 2798.27 2794.82	4 10 100R 150R 200R (8R) (9R) (10R)	I A I A I I I .....	(1s <sup>6</sup> —1p <sup>8</sup> )  1s <sup>6</sup> —1p <sup>6</sup>  1s <sup>6</sup> —2p <sup>6</sup>	
2.16.....	5537.75 5516.77 5505.88 5481.40 5470.64 5457.47 5420.37 5407.43 5341.07 4083.64 4082.95 4079.43 4079.24 4058.94 4055.55 4048.76 4041.37 4035.73 4018.11 3841.08 3834.36 3823.90 3823.51 3809.60 3806.87	5 7 2 4 8 1 10 5 20 12 12 10 12 10 20 15 50R 15 20 10 12 10 20R 10 20R	III III A III II A II A II A III A III A III A I I I I I I I I I I II II II II II I	1d <sup>6</sup> —2p <sup>6</sup>             1d <sup>6</sup> —1f <sup>6</sup> *            1d <sup>6</sup> —1f <sup>6</sup> *	



TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Manganese—Continued					
2. 16—Cont.	3629.74	12	III	Id <sup>6</sup> —3p <sup>6</sup>	
	3623.79	15	III		
	3619.40	15	III		
	3610.30	20	III		
	3608.48	20	II		
	3607.53	20	II		
	3595.11	20	III		
	3586.54	30	II		
	3577.88	40	II		
2. 29.....	4823.52	50	I	Ip <sup>8</sup> —1s <sup>8</sup>	
	4783.43	50	I		
	4754.04	50	I		
	3570.03	20	III	Ip <sup>8</sup> —Id <sup>8</sup>	
	3569.80	40	III		
	3569.50	60	III		
	3548.18	30	III		
	3548.02	40	III		
	3547.79	50	III		
	3532.11	50	III		
	3531.09	50	III		
	3531.83	30	III		
3. 07.....	6021.78	50	III	Ip <sup>6</sup> —2s <sup>6</sup>	
	6016.64	40	III		
	6013.49	30	III		
	4061.74	5	V	Ip <sup>6</sup> —3d <sup>6</sup>	
	4059.40	5	IV		
	4057.96	4	V		
4. 41.....	7326.55	(7)	.....	2p <sup>6</sup> —3s <sup>6</sup>	
	7302.92	(6)			
	7283.80	(6)			
?.....	4766.43	20	III	Id <sup>4</sup> —1f <sup>4</sup>	(24)
	4765.86	10	III		
	4762.37	30	III		
	4761.53	10	III		
	4739.00	8	III		
	4727.46	10	III		
	4709.70	10	III		
	4701.16	3	V		
	4671.69	3	V		
	4502.22	7	III	Id <sup>4</sup> —Id <sup>4</sup>	
	4498.90	7	III		
	4490.08	5	III		
	4472.79	5	III		
	4470.14	6	III		
	4464.68	8	II		
	4453.01	6	III		
	4451.58	15	II		
	4436.36	8	III		
	4414.89	10	II		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Manganese—Continued					
?—Cont....	4312.55	3	II	1d <sup>4</sup> —1p' <sup>4</sup>	
	4284.08	(5)	...		
	4281.10	6	II		
	4265.92	6	II		
	4257.66	5	II		
	4239.73	5	II		
	4235.30	8	I		
	4235.14	6	II		
Iron, Fe, I.P.?					(25)
O.O.....	5254.96	1	IA)	(1d <sup>s</sup> —1d' <sup>7</sup> )	
	5250.21	1	IA		
	5247.05	1	IA		
	5225.53	1	IA		
	5204.59	2	IA		
	5168.90	4	IA		
	5166.29	4	IA		
	5110.41	10	I		
	4489.74	3	IA)	(1d <sup>s</sup> —1f' <sup>7</sup> )	
	4482.18	4	I		
	4461.65	8	I		
	4445.43	(1)	IA		
	4427.31	10	I		
	4375.93	9	I		
	4347.24	(1)	.....)		
	4291.47	4	IA)	(1d <sup>s</sup> —1p' <sup>7</sup> )	
	4258.38	2	IA		
	4232.72	1	IA		
	4216.18	8	I		
	4206.70	3	IA		
	4199.98	1	IA		
	4134.34	(1)	.....)		
	3930.30	25R	I	1d <sup>s</sup> —1d' <sup>s</sup>	
	3927.92	30R	I		
	3922.92	25R	I		
	3920.26	20R	I		
	3906.48	8	I		
	3899.71	30R	I		
	3895.66	25R	I		
	3886.29	40R	I		
	3878.58	100R	II		
	3859.91	300R	I		
	3856.37	50R	IA		
	3824.44	50R	IA)		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes	
Iron—Continued						
0.0—Cont.	3748.26	60R	I A	rd <sup>s</sup> —rf' <sup>s</sup>		
	3745.90	40r	I A			
	3745.56	100R	I			
	3737.14	150R	I			
	3733.32	40r	I A			
	3722.57	50r	I A			
	3719.94	250R	I			
	3707.83	20	I			
	3705.57	100r	I			
	3683.06	10	I A			
	3679.92	40r	I A			
	3649.31	5	I A			
	3526.07	20	I	rd <sup>s</sup> —rp' <sup>s</sup>		
	3497.84	40	I			
	3490.58	100r	I			
	3476.71	40	I			
	3475.45	70r	I			
	3465.86	60r	I			
	3443.88	50r	I			
	3440.99	75R	I			
	3440.61	150R	I			
0.94.....	8075.16	(1)	.....	(rf <sup>s</sup> —rd' <sup>7</sup> )		
	8047.63	(1)				
	7912.89	(1)				
	6625.04	(1)	.....	(rf <sup>s</sup> —rf' <sup>7</sup> )		
	6574.24	(2)				
	6498.95	(2)				
	6462.74	(4)				
	6400.33	(2)				
	6358.68	3	I A	.....		
	6280.62	2	I A			
	6221.70	(0)				
	6120.25	(0)	.....	(rf <sup>s</sup> —rp' <sup>7</sup> )		
	5958.20	(2)				
	5956.70	(3)				
	5949.35	(2)	I	rf <sup>s</sup> —rd' <sup>s</sup>		
	5506.78	18				
	5501.47	12				
	5497.52	15				
	5455.62	40				
	5446.92	40				
	5434.53	30				
	5429.70	40				
	5405.78	40				
	5397.14	40				
	5371.50	50				
	5328.04	50				
	5269.54	60				

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Iron—Continued					
0.94—Cont.	5151.92	4	I	1f <sup>s</sup> —1f' <sup>s</sup>	
	5150.84	6	I		
	5142.93	6	I		
	5127.36	5	I		
	5123.72	6	I		
	5107.45	6	I		
	5083.34	7	I		
	5079.74	4	I		
	5051.64	10	I		
	5041.08	7	I		
	5012.07	12	I		
	4994.13	8	I		
	4939.69	4	I		
	4239.85	2	III	(1f <sup>s</sup> —1f' <sup>s</sup> )	
	4177.60	4	II A		
	4152.18	4	II A		
	4139.93	2	II A		
	4100.75	3	II A		
	4092.51	(1)	.....		
	3940.89	5	II	1f <sup>s</sup> —2d' <sup>s</sup>	
	3917.19	8	II		
	3898.01	10	II		
	3887.05	15	I		
	3878.02	60	II		
	3872.50	60	II		
	3865.53	30	II		
	3849.97	40	II		
	3840.44	80r	II		
	3834.23	100r	II		
	3825.89	200R	II	(1f <sup>s</sup> —1p' <sup>s</sup> )	
	3820.43	250R	II		
	3876.04	4	III		
	3850.82	12	II		
	3814.53	5	III A		
	3812.97	40	II		
	3790.09	12	II		
	3786.68	8	III		
	3799.55	50	II	1f <sup>s</sup> —2f' <sup>s</sup>	
	3798.51	40	II		
	3795.00	60	II		
	3787.88	50	II		
	3767.19	80r	II		
	3763.79	100r	II		
	3758.23	150R	II		
	3749.49	200R	II		
	3743.36	20	II A		
	3734.87	300r	II		
	3727.62	50r	II		
	3709.25	75r	II		
	3687.46	40r	I		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Iron—Continued					
0.94—Cont.	3647.84	100R	I	If <sup>3</sup> —1g <sup>5</sup> *	
	3631.46	125R	I		
	3618.77	125R	I		
	3608.87	100r	I		
	3581.20	250R	I		
	3570.10	100R	I	(If <sup>3</sup> —1g <sup>5</sup> )	
	3565.38	60r	II		
	3558.52	30	II		
1.54.....	7362.32	(0)	.....	(If <sup>3</sup> —1d <sup>5</sup> )	
	7180.02	(1)			
	6739.54	(1)	.....	(If <sup>3</sup> —1f <sup>5</sup> )	
	6710.31	(1)			
	6581.22	(2)			
	5853.18	(1)	.....	(If <sup>3</sup> —1p <sup>5</sup> )	
	5341.03	20	II	If <sup>3</sup> —1d <sup>3</sup>	
	5328.53	15	II		
	5270.36	30	II		
	5227.19	40	II		
	5167.49	40	II		
	5332.91	4	I	If <sup>3</sup> —1f <sup>3</sup>	
	5307.36	2	III		
	5216.28	10	II		
	5194.95	10	I		
	5171.60	20	II		
	5107.65	8	II	.....	(If <sup>3</sup> —2d <sup>5</sup> )
	5041.76	10	III		
	4798.74	(1)	.....		
	4772.82	3	III	I	(If <sup>3</sup> —1p <sup>3</sup> )
	4733.59	4	I		
	4765.48	(1)	.....	(If <sup>3</sup> —2f <sup>5</sup> )	
	4680.30	(2)	.....		
	4654.50	5	II		
	4632.92	2	III?		
	4602.94	9	I		
	4602.01	(2)	.....	.....	(If <sup>3</sup> —1g <sup>5</sup> )
	4592.66	5	I		
	4547.02	(2)	.....		
	4531.15	8	II		
	4415.13	20	II		
	4404.75	30	II	III A	
	4383.55	45r	II		
	4367.91	2	III A		
	4337.95	10	II		
	4294.13	15	II		
	4291.47	4	I A	III	
	4229.75	(1)	III		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes			
Iron—Continued								
1. 54—Cont.	4325.77	35	II	1f <sup>3</sup> —1g' <sup>3</sup>				
	4307.91	35	II					
	4271.76	35	II					
	4250.79	25	II					
	4202.03	30	I					
	4147.68	10	III	1f <sup>3</sup> —2f' <sup>3</sup>				
	4143.87	30	I					
	4132.06	25	II					
	4071.75	40	II					
	4063.60	45	II					
	4045.82	60r	II	1f <sup>3</sup> —2d' <sup>3</sup>				
	4005.25	25	II					
	3969.26	30	II					
	3966.07	10	III					
	3902.95	20	II					
	3888.52	20	II	1f <sup>3</sup> —2d' <sup>3</sup>				
	3841.05	80r	II					
	3827.83	75r	II					
	3815.84	100r	II					
2. 19.....	8824.18	(6)	.....	1p <sup>5</sup> —1p' <sup>5</sup>				
	8688.58	(7)						
	8661.85	(6)						
	8514.01	(3)						
	8468.35	(7)						
	8387.74	(8)	III	1p <sup>5</sup> —2d' <sup>5</sup>				
	8327.04	(8)						
	6430.86	10						
	6335.34	10						
	6297.80	5		1p <sup>5</sup> —2p' <sup>5</sup>				
	6265.14	6	III					
	6219.29	6	III					
	6213.44	5	III					
	6173.35	3	III					
	6151.63	(2)	...	(1p <sup>5</sup> —1p' <sup>5</sup> )				
	6137.00	(2)	...					
	6240.65	(2)	...					
	6163.56	(1)	...					
	6082.71	(1)	...					
	5250.65	6	IV	1p <sup>5</sup> —2p' <sup>5</sup>				
	5202.34	8	IV					
	5198.72	4	IV					
	5142.54	(3)	...					
	5131.48	(2)	...					
	5098.71	8	IV	(1p <sup>5</sup> —2d' <sup>5</sup> )				
	5079.23	6	IV					
	4889.02	(1)	.....					
	4817.78	(1)						
	4771.70	(1)						
	4745.12	(1)						

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Iron—Continued					
2. 19—Cont.	4528.62	18	II		
	4494.57	12	II		
	4482.26	6	I		
	4459.13	10	III		
	4447.73	9	III	1p <sup>s</sup> —3d <sup>'s</sup>	
	4442.35	12	III		
	4430.62	6	III		
	4408.42	6	III		
	4407.72	5	III		
	4352.74	9	III	1p <sup>s</sup> —1s <sup>'s</sup>	
	4315.09	10	III		
	4282.41	12	III		
2. 46.....	4299.25	18	III		
	4271.17	20	III		
	4260.49	35	III		
	4250.13	25	III		
	4235.95	25	III		
	4233.61	18	III	1d <sup>'7</sup> —md <sup>7</sup>	
	4222.23	12	III		
	4210.36	15	III		
	4198.31	20	III		
	4191.45	15	III		
	4187.81	20	III		
2. 84.....	4187.05	20	III		
	5044.24	(2)	...		
	5006.13	20	III		
	4985.56	7	V		
	4957.61	60	III		
	4957.31	20	III		
	4938.82	10	IV		
	4920.52	60	III		
	4919.01	30	III	1f <sup>'7</sup> —md <sup>7</sup>	
	4903.32	12	III		
	4891.51	50	III		
	4890.77	25	III		
	4878.23	12	III		
	4872.15	20	III		
	4871.33	25	III		
	4859.76	15	III		
	4579.34	(1h)	.....	(1f <sup>'7</sup> —md <sup>s</sup> )	
	4554.47	(1)			
	4525.88	(1)			
	4495.39	(1h)			
2. 96.....	5281.80	10	IV		
	5266.57	30	IV		
	5232.96	40	III		
	5226.88	15	IV		
	5192.36	30	IV	1p <sup>'7</sup> —md <sup>7</sup>	
	5191.47	20	IV		
	5139.48	20	IV		
	5139.27	10	IV		
	5068.78	10	V		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Iron—Continued					
2.96—Cont.	4800.14	(1)	V	(1p' <sup>7</sup> —md <sup>5</sup> )	
	4787.84	(1)			
	4768.40	3n			
	4759.98	(rh)			
	4726.17	(rh)			
3.25.....	5393.19	10	IV	rd' <sup>5</sup> —md <sup>5</sup>	
	5339.95	12	V		
	5324.20	30	IV		
	5302.31	10	V		
	5283.63	18	IV		
	5273.18	(3)	...		
	5263.32	8	V		
	5253.48	(2)	...		
	5229.84	5n	V		
	5217.40	5	V		
	5215.20	6	IV		
	5208.61	7	IV		
	4736.79	12	II?		
	4707.29	8	IV		
	4668.15	6	IV		
	4654.64	5	V		
	4637.52	3	IV		
	4625.06	3	IV		
	4613.22	2n	V		
	4607.66	3n	V		
	4598.14	(2)	...		
	4568.79	(1)	...		
	4565.68	(2)	...		
3.38.....	5784.69	(1)	...	rf' <sup>5</sup> —md <sup>5</sup>	
	5712.15	(2)	...		
	5709.40	10	IV		
	5658.84	10	IV		
	5658.54	(1)	...		
	5624.56	10	IV		
	5615.66	50	IV		
	5602.96	10	IV		
	5586.77	40	IV		
	5576.10	10	IV		
	5572.86	30	IV		
	5569.63	20	IV		
	5039.27	(2)	...		
	5002.82	(2)	...		
	4966.10	8	V		
	4950.12	(1)	...		
	4946.40	4	IV		
	4910.03	(2)	...	rf' <sup>5</sup> —mf <sup>5</sup>	*
	4907.74	(1)	...		
	4882.17	(2)	...		
	4863.65	(2)	...		
	4855.60	(2)	...		
	4843.16	(2)	...		
	4838.52	(2)	...		



TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Iron—Continued					
3.64.....	6411.67	12	IV	1p' <sup>s</sup> —md <sup>s</sup>	
	6408.04	8	V		
	6400.02	25	III		
	6336.84	12	V		
	6302.51	6	V		
	6301.52	15	IV		
	6246.34	15	V		
	6232.67	5	V		
	6141.74	4	V		
4.13.....	7207.41	3	V	2d' <sup>s</sup> —mf <sup>s</sup>	
	7187.35	5	V		
	7164.49	(9)	...		
	7130.96	(10)	...		
	7090.41	(6)	...		
	7038.23	(4)	...		
	7022.98	(5)	...		
	7016.30	(9)	...		
	6999.92	(4)	...		
4.23.....	7710.40	(3)	.....	2f' <sup>s</sup> —mf <sup>s</sup>	
	7661.24	(3)			
	7568.94	(4)			
	7511.06	(9)			
	7495.12	(8)			
	7491.68	(2)			
	7445.80	(9)			
	7411.21	(8)			
	7389.43	(7)			
	7311.13	(4)			
	7306.60	(2)			
	7293.01	(6)			
	7288.79	(4)			
4.35.....	8497.00	(2)	.....	1g' <sup>s</sup> —mf <sup>s</sup>	
	8401.42	(1)			
	8248.09	(3)			
	8232.33	(2)			
	8220.42	(7)			
	8207.85	(1)			
	8179.03	(1)			
	8085.22	(5)			
	8046.08	(5)			
	7999.00	(6)			
4.40.....	7937.19	(9)			
	8360.79	(2)	.....	(1g' <sup>s</sup> —mf <sup>s</sup> )	
	8339.41	(4)			
	8331.94	(6)			

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Iron, $Fe^+$ , I.P.?					(18)
O. O. ....	2625.68	(9)	...	$Id^6 - Id'^6$ *	
	2611.89	(9)	...		
	2599.40	(10)	...		
	2585.89	(10)	IV		
	2410.53	(8)	...	$Id^6 - If'^6$ *	
	2404.89	(8)	...		
	2395.63	(8)	...		
	2382.04	(10)	II		
	2373.73	(7)	II		
	2343.49	(7)	.....	$Id^6 - Ip'^6$ *	
	2332.80	(6)			
	2327.39	(6)			
O. 33. ....	2519.09	(6)	.....	$If^4 - If'^6$ (* )	
	2493.29	(10)			
	2366.59	(4)	...	$If^4 - If'^4$ *	
	2362.04	(4)	...		
	2360.00	(5)	...		
	2354.88	3	II		
	2375.19	(6)	...	$If^4 - Id'^4$ *	
	2368.60	(6)	III		
	2360.31	(5)	...		
	2348.12	(6)	...		
I. 04. ....	3281.30	(2)	.....	$Id^4 - Id'^6$ (* )	
	3277.36	(3)			
	3255.89	(2)			
	2979.36	(3)	...	$Id^4 - If'^6$ (* )	
	2970.52	(3)	V		
	2953.78	5	V		
	2926.58	(6)	V		
	2755.74	(9)	.....	$Id^4 - If'^4$ *	
	2749.33	(10)			
	2746.49	(8)			
	2743.20	(8)			
	2746.99	(7)	III	$Id^4 - Id'^4$ *	
	2739.55	(10)	...		
	2736.97	(7)	...		
	2714.42	(8)	...		
	2591.55	(7)	.....	$Id^4 - Ip'^4$ *	
	2582.59	(8)			
	2562.54	(9)			
I. 69. ....	3227.76	(7)	.....	$Ip^4 - Id'^4$ *	
	3213.32	(5)			
	3210.46	(4)			

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Iron—Continued					
1.69—Cont.	3002.65 2985.55 2984.83	(7) (6) (8)	V V IV	$1p^4-1p'^4$	
2.68.....	4369.41 4296.56 4258.16 4178.87	.. (6) .. (6)	.....	$2p^4-1f'^4$	
	4416.81 4385.39 4351.77 4303.18 4233.16 4173.48 4128.74	(4) .. (6) (4) (8) (6) ..	.....	$2p^4-1d'^4$	
2.82.....	4666.75 4629.33 4582.84 4555.90 4534.17 4520.24 4515.33 4491.41 4489.21 4472.93	.. (4) .. (6) .. (6) (6) (4) (4) ..	.....	$2f^4-1f'^4$	
	4620.52 4583.84 4576.31 4549.48 4541.52 4522.64 4508.29	.. (8) (4) (4) .. (6) (8)	.....	$2f^4-1d'^4$	
2.88.....	6516.08 6432.60 6369.48	(4) .. ..	.....	$1s^6-1d'^6$	
	5284.11 5256.94	.. ..	.....	$1s^6-1f'^6$	
	5160.03 5018.44 4923.92	(8) (8) (10)	.....	$1s^6-1p'^6$	
	4731.49 4656.98	(1) ..	.....	$(1s^6-1d'^4)$	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Iron—Continued					
3.18.....	5425.26	..	.....	$1g^4-1f'^4$	*
	5325.56	(4)			
	5316.62	(8)			
	5276.01	(6)			
	5234.62	(4)			
	5197.56	(4)			
	5362.87	..			
	5316.78	..			
	5264.81	(1)			
	5264.81	(1)			
3.88.....	7711.76	(1)	.....	$2d^4-1d'^4$	
	7655.47	..			
	7515.89	..			
	7449.36	..			
	7310.23	..			
	7308.09	..			
	7224.48	..			
	7222.42	..			
	6456.39	(6)			
	6416.92	(2)			
	6407.30	..			
	6247.56	(4)			
	6239.95	..			
	6238.39	(4)			
	6149.25	(4)			
	6147.85	(6?)			
Cobalt, Co, I.P. ?					
(32)					
0.0.....	3602.08	40 R	II	$1f^4-1f''^4$	*
	3594.87	50 R	II		
	3575.36	60 R	II		
	3526.86	100 R	II		
	3474.02	100 R	II		
	3533.36	25 R	I	$1f^4-1g^4$	*
	3529.04	30 R	I		
	3513.48	50 R	II		
	3465.80	100 R	II		
	3510.42	30 R	I		
	3502.63	20 R	I	$1f^4-1d^4$	*
	3455.24	25 R	I		
	3442.92	20 R	II		
	3431.58	50 R	II		
	3412.64	80 R	II		
	3159.66	10	II	$1f^4-1d'^4$	*
	3149.30	10	II		
	3139.94	12	II		
	3121.41	10	II		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Cobalt—Continued					
0.41.....	4076.13	4	I A	$1f'^4 - 1f''^4$ *	
	4058.19	8	I		
	4020.90	20	I		
	4019.30	5	I		
	3987.12	6	I		
	3952.33	8	I	$1f'^4 - 1g^4$ *	
	3978.66	10	I		
	3041.74	20	II		
	3006.30	10	I		
	3876.84	20	I		
	3808.11	10	I	$1f'^4 - 1d^4$ *	
	3957.94	15	II		
	3894.98	60	II		
	3881.91	25	I		
	3873.91	40	II		
	3873.12	60	II	$1f'^4 - 1d'^4$ *	
	3585.16	25 R	I		
	3574.96	25 r	I		
	3560.90	20 r	I		
	3523.44	25 r	I		
1.61.....	3512.64	60 R	II	$1p^4 - 1d^4$ *	
	3506.32	80 R	II		
	3502.28	100 R	II		
	6450.23	80	I		
	6282.64	40	I	$1p^4 - 1d'$ *	
	6188.98	10	II A		
	6093.14	10	I		
	5483.34	40	I	$1p^4 - 1d'^4$ *	
	5369.58	20	I		
	5331.45	15	II		
1.81.....	5301.04	15	II		
	5237.91	15	II		
	5230.20	25	II		
	7084.97	15	I	$1p'^4 - 1d^4$ *	
	7052.85	8	I		
	6872.38	10	I		
	6814.95	15	I		
	6771.05	20	I		
	5935.37	5	III	$1p'^4 - 1d'^4$	
	5922.35	(1u)	...		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Lanthanum, $La^+$ , I.P.?					(33)
0.0.....	4141.71 4031.70 3988.53 3921.54 3886.34  3380.88 3344.55 3337.50 3303.10 3265.66	(10) (20) (30) (10) (15)  (10) (7) (15) (5) (4)	  		

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Ionized Nitrogen—Continued					(34)
15.2—Cont.	4674.98 4667.28 4654.57	(2) (2) (2)	.....	(mP—mp <sup>3</sup> )	
	3995.00 3437.16	(10) (6)	..... .....	mP—mS mP—nS	
17.2.....	4810.27 4803.30 4793.66 4788.13 4781.21 4779.72 4774.24	(2) (6) (2) (5) (2) (4) (2)	.....	md <sup>3</sup> —md <sup>3</sup>	
	3331.32 3330.30 3328.79	(3) (2) (4)	.....	md <sup>3</sup> —np <sup>3</sup> *	
17.7.....	5961.25 5952.57 5941.90 5940.42 5931.96 5928.02	(0) (3) (8) (2) (7) (4)	.....	mp <sup>3</sup> —md <sup>3</sup>	
	3856.07 3855.08 3847.38 3842.20 3838.39 3829.80	(3) (3) (2) (2) (4) (3)	.....	mp <sup>3</sup> —np <sup>3</sup>	
Phosphorus, P, I.P. 13.3					(30)
0.0.....	2034.02 2032.98 2024.98 2023.98	(7) (6) (6) (7)	.....	1d—1d'	
0.38.....	2154.77 2153.63	(7) (6)	.....	1S—1d'	
1.25.....	2555.68 2554.02 2536.38 2534.75	(8) (9) (10) (8)	.....	1D—1d'	
	2149.81 2136.79 2136.10	(8) (8) (6)	.....	1D—1P'	

TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes	
Arsenic, As, I.P. 11.7					(31)	
O. O. ....	2808.72	(4R)	.....	1d-1d'		
	2860.43	(6R)				
	2780.20	(8R)				
	2744.99	(6R)				
	2349.84	(10R)	.....	1d-1p'		
	2288.12	(10R)				
O. 2I. ....	2381.20	(4R)	.....	1S-1p'		
	2363.04	(2)				
O. 4I. ....	3119.57	(4)	.....	1D-1d'		
	3075.32	(2)				
	3032.84	(4)				
	2990.99	(2)				
Antimony, Sb, I.P. 8.7					(31)	
O. O. ....	6097.60	(2)	.....	(1d <sub>2</sub> -1D <sub>2</sub> )		
	2311.50	(6R)		1d <sub>2</sub> - $\alpha$		
	2175.88	(5R)		1d <sub>2</sub> - $\beta$		
	2127.46	(3)		1d <sub>2</sub> - $\gamma$		
	2023.86	(4)		1d <sub>2</sub> - $\epsilon$		
I. 05. ....	2877.92	(5R)	.....	1d <sub>1</sub> - $\alpha$		
	2670.66	(5R)		1d <sub>1</sub> - $\beta$		
	2598.08	(6R)		1d <sub>1</sub> - $\gamma$		
	2445.53	(4R)		1d <sub>1</sub> - $\epsilon$		
I. 22. ....	2769.94	(5R)	.....	1S- $\beta$		
	2528.54	(6R)		1S- $\epsilon$		
I. 98. ....	3722.79	(3)	.....	1D <sub>2</sub> - $\alpha$		
	3383.15	(3)		1D <sub>2</sub> - $\beta$		
	3267.50	(4R)		1D <sub>2</sub> - $\gamma$		
	3029.82	(4R)		1D <sub>2</sub> - $\epsilon$		
2. 24. ....	4033.56	(3)	.....	1D <sub>1</sub> - $\alpha$		
	3637.83	(3)		1D <sub>1</sub> - $\beta$		
	3504.45	(2)		1D <sub>1</sub> - $\gamma$		
	3232.54	(4R)		1D <sub>1</sub> - $\epsilon$		
Bismuth, Bi, I.P. 7.7					(31)	
O. O. ....	8754.88	(2)	.....	(1d <sub>2</sub> -1d <sub>1</sub> )		
	6476.24	(3)		(1d <sub>2</sub> -1D <sub>2</sub> )		
	4615.53	(1)		(1d <sub>2</sub> -1D <sub>1</sub> )		
	4615.09	(1)				



TABLE I—Continued

E.P.	$\lambda$	Int.	Class	Type	Notes
Bismuth—Continued					
0.0—Cont.	3067.69	(10R)	.....	1d <sub>2</sub> — $\alpha$	
	2276.57	(8R)	.....	1d <sub>2</sub> — $\delta$	
	2230.63	(10R)	.....	1d <sub>2</sub> — $\epsilon$	
1.41.....	4722.54	(10R)	.....	1d <sub>1</sub> — $\alpha$	
	3076.61	(2)	.....	1d <sub>1</sub> — $\delta$	
	2993.34	(8R)	.....	1d <sub>1</sub> — $\epsilon$	
	2989.03	(8R)	.....	1d <sub>1</sub> — $\zeta$	
	2897.97	(10R)	.....	1d <sub>1</sub> — $\eta$	
1.92.....	3510.88	(4R)	.....	1D <sub>2</sub> — $\delta$	
	3397.17	(4R)	.....	1D <sub>2</sub> — $\zeta$	
2.68.....	4492.99	(2) \	.....	1D <sub>1</sub> — $\delta$	
	4492.62	(2) }	.....		
	4308.54	(4) \	.....	1D <sub>1</sub> — $\zeta$	
	4308.18	(4) }	.....		
	4121.86	(6) \	.....	1D <sub>1</sub> — $\eta$	
	4121.54	(6) }	.....		

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Symbol	Element	Page	Symbol	Element	Page
<i>Ag</i> .....	Silver	235	<i>Fe</i> .....	Iron	266
<i>Al</i> .....	Aluminum	241	<i>Fe</i> <sup>+</sup> .....	Ionized iron	274
<i>Al</i> <sup>+</sup> .....	Ionized aluminum	241	<i>Ga</i> .....	Gallium	242
<i>Al</i> <sup>++</sup> .....	Doubly ionized aluminum	242	<i>H</i> .....	Hydrogen	231
<i>As</i> .....	Arsenic	280	<i>He</i> .....	Helium	231
<i>Au</i> .....	Gold	235	<i>He</i> <sup>+</sup> .....	Ionized helium	232
<i>B</i> .....	Boron	241	<i>Hg</i> .....	Mercury	241
<i>Ba</i> .....	Barium	239	<i>In</i> .....	Indium	242
<i>Ba</i> <sup>+</sup> .....	Ionized barium	240	<i>K</i> .....	Potassium	233
<i>Be</i> .....	Beryllium	235	<i>La</i> <sup>+</sup> .....	Ionized lanthanum	278
<i>Be</i> <sup>+</sup> .....	Ionized beryllium	235	<i>Li</i> .....	Lithium	233
<i>Bi</i> .....	Bismuth	280	<i>Mg</i> .....	Magnesium	235
<i>C</i> <sup>+</sup> .....	Ionized carbon	242	<i>Mg</i> <sup>+</sup> .....	Ionized magnesium	236
<i>Ca</i> .....	Calcium	236	<i>Mn</i> .....	Manganese	264
<i>Ca</i> <sup>+</sup> .....	Ionized calcium	238	<i>Mo</i> .....	Molybdenum	264
<i>Cd</i> .....	Cadmium	240	<i>N</i> <sup>+</sup> .....	Ionized nitrogen	278
<i>Cd</i> <sup>+</sup> .....	Ionized cadmium	241	<i>Na</i> .....	Sodium	233
<i>Co</i> .....	Cobalt	276	<i>Ne</i> .....	Neon	232
<i>Cr</i> .....	Chromium	261	<i>O</i> .....	Oxygen	243
<i>Cs</i> .....	Caesium	234	<i>P</i> .....	Phosphorus	279
<i>Cu</i> .....	Copper	234	<i>Pb</i> .....	Lead	243
			<i>Rb</i> .....	Rubidium	234

\*In this table the elements have been arranged in the alphabetical order of their chemical symbols.

## INDEX TO TABLE I—Continued

Symbol	Element	Page	Symbol	Element	Page
S.....	Sulphur	244	Ti+.....	Ionized titanium	254
Sb.....	Antimony	280	Tl.....	Thallium	242
Sc.....	Scandium	244	V.....	Vanadium	258
Sc+.....	Ionized scandium	245	Y.....	Yttrium	246
Se.....	Selenium	244	Y+.....	Ionized yttrium	247
Si++++..	Treble ionized silicon	243	Zn.....	Zinc	240
Sr.....	Strontium	238	Zn+.....	Ionized zinc	240
Sr+.....	Ionized strontium	239	Zr.....	Zirconium	257
Ti.....	Titanium	247	Zr+.....	Ionized zirconium	257

## NOTES TO TABLE I

1. Lyman, *Physical Review*, **21**, 202, 1923; *Science*, **59**, 422, 1924.
2. Paschen, *Annalen der Physik*, **60**, 405, 1919. Only lines of intensity greater than 10 (or equal to 10 in the ultra-violet) are listed. The ultimate lines are unknown, and the ionization potential is taken from the electrical determination by G. Hertz (*Zeitschrift für Physik*, **18**, 307, 1923). Lines originating in the four 1s-levels, which have an extreme range of 0.22 volts, are listed together for convenience, and similarly for the 2p-terms.
3. V. Thorsen, *Die Naturwissenschaften*, **11**, 501, 1923.
4. Back, *Annalen der Physik*, **70**, 347, 1923. The E.P. for the triplets is probably less than for the singlets, by analogy with other elements in the group.
5. I.P. estimated by Saunders, *Science*, **59**, 51, 1924.
6. The lines not belonging to the ordinary series are as classified by Russell and Saunders (in press).
7. Paschen, *op. cit.*, **71**, 538, 1923. Russell, *Nature*, **113**, 163, 1924.
8. Paschen, *op. cit.*, **71**, 151, 1923.
9. Fowler, *Proceedings of the Royal Society, A*, **105**, 299, 1924.
10. Fowler, *ibid.*, **103**, 413, 1923.
11. Thorsen, *Die Naturwissenschaften*, **11**, 78, 1923; Grottrian, *Zeitschrift für Physik*, **18**, 169, 1923.
12. Hopfield, *Astrophysical Journal*, **59**, 114, 1924.
13. Hopfield, *Nature*, **112**, 437, 1923.
14. Catalan, *Anales Soc. Españ. Fis. y Quim.* **20**, 606, 1922, and **21**, 464, 1923. Supplemented by the writer (unpublished).
15. The lowest term cannot be a  $p^2$ -term, as Catalan supposes, for it combines with doublet terms in adjacent series on both sides. It may be a d- or f-term. The Zeeman separations in sun-spots (the only ones available) favor the former, and the terms have been provisionally classified accordingly.
16. The important enhanced line  $\lambda$  4247 probably belongs to the singlet system. From its behavior in the furnace it is probable that it originates from a higher level than the ultra-violet triplets.
17. C. C. and H. H. Kiess, *Proceedings of the Washington Academy of Sciences*, **13**, 270, 1923, and unpublished investigations by the writer.
18. Unpublished investigations by the writer.

19. W. F. Meggers, *Journal of the Washington Academy of Sciences*, **13**, 317, 1923. Supplemented by unpublished investigations communicated by Dr. Meggers.
20. H. Gieseler, *Zeitschrift für Physik*, **22**, 228, 1924.
21. The three faint lines appear in the sun-spot spectrum, and agree exactly with the predicted positions of members of this group.
22. C. C. Kiess, *Scientific Papers of the Bureau of Standards*, **19**, 113 (No. 474), 1923.
23. M. A. Catalan, *Philosophical Transactions*, A, **223**, 127, 1922.
24. Back, *Zeitschrift für Physik*, **15**, 234, 1923. The excitation potential for these groups is shown by their temperature behavior to be fairly high.
25. F. M. Walters, Jr., *Journal of the Optical Society of America*, **8**, 245, 1924. Supplemented by unpublished work of Dr. Walters and of the writer.
26. From unpublished data communicated by Dr. Meggers.
27. From unpublished data communicated by Dr. Kiess.
28. The wave-lengths here given are from unpublished measures made at the Bureau of Standards by Dr. Meggers, and kindly communicated by him.
29. Unpublished Zeeman observations by Moore confirm the classification of the lowest level as a D-term.
30. Miss Saltmarsh, *Philosophical Magazine*, **47**, 874, 1924. Energy-levels and designation of terms added by the writer in analogy with those for *As*, *Sb*, and *Bi*.
31. Ruark, Mohler, Foote, and Chenault, *Scientific Papers, Bureau of Standards*, No. 490, 1924. I.P.'s from critical potentials and uncertain by 0.5 volt.
32. Walters, *Journal of the Washington Academy of Sciences*, **14**, 407, 1924.
33. Gouschmidt, *Die Naturwissenschaften*, **41**, 851, 1924. The d<sup>3</sup>-term is assumed to be the lowest, by analogy with *Sc*<sup>+</sup> and *Y*<sup>+</sup>. Many other terms are present, with peculiar Zeeman effects.
34. Fowler, *Proceedings of the Royal Society*, A, **107**, 31, 1925. I.P. estimated by Miss Payne, *Harvard College Observatory Circular*, No. 256, 1924.

MOUNT WILSON OBSERVATORY

May 12, 1924

Additions have been made to date.

PRINCETON UNIVERSITY OBSERVATORY

January 27, 1925

## SOME RELATIONS BETWEEN MAGNITUDE SCALES<sup>1</sup>

By FREDERICK H. SEARES

### ABSTRACT

*Systematic corrections to various systems of magnitudes.*—The principal purpose is the reduction of the Harvard visual scale to the Mount Wilson photovisual system. Incidentally, it is necessary to discuss the relation of this system to the photographic and photovisual magnitudes of King and the visual results of the Potsdam Durchmusterung. A direct comparison of Polar Sequence measures gives the values of the color equation, which seems to depend on the magnitude, and the scale correction for the Harvard scale between  $m=4$  and 13 (Table II and [2]). King's extra-focal measures define scales which are sensibly parallel to *MW* and *H* results for  $m < 4$  (Tables III-V) and extend the series of color coefficients, but leave the zero-point correction to the Harvard visual magnitudes of bright stars rather uncertain. The result, however, is confirmed by comparing, in the critical region, the *PD* and *HR* and the *PD* and *MW* systems. The scale corrections for Harvard are approximately  $-0.08$  to  $m=5$ ;  $+0.07$  ( $m-6$ ) for  $m=5$  to 9.5; and  $-0.08$  ( $m-12.5$ ) for  $m=9.5$  to 13. The changes in the run in the differences correspond to the division-points between measures made with the 2-inch, 4-inch, and 12-inch meridian photometers. The comparison of Harvard and Potsdam is based mainly on the data of Müller and Kempf and leads finally to the scale and color corrections for the *PD* (Table VI), confirmed in part by direct comparison of *PD* and *MW* magnitudes for Polar Sequence stars.

The problem of reducing existing photometric data to a homogeneous system is very complicated and cannot be satisfactorily accomplished at present because of lack of information as to scale errors and color equations. The difficulties in the case of visual magnitudes are especially troublesome, since the color equation frequently varies with the brightness of the stars.<sup>2</sup> The following pages discuss the relations between certain photometric systems, chiefly Harvard, Potsdam, and Mount Wilson, for which details can now be traced with some definiteness.

Satisfactory photographic standards are now available in the magnitudes of stars near the Pole, collected and combined by the Commission on Stellar Photometry.<sup>3</sup> The state of affairs with respect to visual standards is less satisfactory. Because of certain definite advantages, the photovisual scale is much to be preferred

<sup>1</sup> Contributions from the Mount Wilson Observatory, No. 288.

<sup>2</sup> See, for example, *Harvard Annals*, 64, 116 ff.; *Potsdam Publikationen*, 17, Introduction, 1917; *Göttingen Aktinometrie*, B, 24, 1912.

<sup>3</sup> *Trans. Internat. Astron. Union*, 1, 69, 1922. For brevity this report of the Commission is referred to in the text simply as the *Report*.

as an ultimate standard to which all other systems of visual and photovisual magnitudes shall be reduced. Briefly, these advantages are, first, the general advantages of a photographic method over that of visual photometry. Second, the freedom, at least comparatively speaking, of the color equation from changes depending on the brightness of the stars. And third, the slight dependence of the color equation on the instrument. As an illustration of the last point, photovisual magnitudes determined at Mount Wilson with the 10-inch Cooke refractor and the 60-inch reflector are sensibly identical in color value, although measures of photographic brightness with these instruments show a relative equation amounting to one-fifth the color-index.<sup>1</sup> Even large differences in the photographic plate and the filter make surprisingly little difference. Thus Parkhurst, for the photovisual magnitudes of the Yerkes actinometry, used a trichromatic plate and a visual-luminosity filter covering the interval  $\lambda 4000\text{--}\lambda 6200$ . At Mount Wilson we have used an isochromatic plate and a filter which limit the region of spectral sensibility to  $\lambda 5000\text{--}\lambda 5900$ , and yet the two color systems are practically the same.

#### I. THE MOUNT WILSON PHOTOVISUAL SYSTEM

The only photovisual scale covering a wide interval in brightness is that defined by the Mount Wilson magnitudes of stars near the Pole. These were determined with the 10-inch Cooke refractor and the 60-inch reflector and are given in final form in the *Report*. The results have not been confirmed by published data obtained elsewhere, but for the following reasons there can be little doubt of their substantial accuracy. (1) The methods used were those employed in establishing the photographic scale, which is known to be free from sensible systematic error. (2) Measures with the Cooke refractor between  $m=6.5$  and  $12.5$  agree with those made with the reflector.<sup>2</sup> (3) Reduced to photovisual values by means of color-indices determined by the method of exposure ratios, the photo-

<sup>1</sup> Seares and Humason, *Mt. Wilson Contr.*, No. 234; *Astrophysical Journal*, 56, 84, 1922.

<sup>2</sup> See columns 17 and 18, Table I, of the *Report*.

graphic magnitudes define a photovisual scale which agrees closely with that established directly. The differences from  $m = 3.0$  to 17.4 are the same as those shown in Table III of the *Report*. The maxi-

TABLE I  
PHOTOVISUAL AND HARVARD VISUAL SCALES, POLAR SEQUENCE

Group	No.	MW $P_v$	Harv. $V_{is.}$	C	MW-HV Corr. for Sc.	Final Color Corr.	Scale Diff. $P_v$ -HV	Residuals
I.....	1s	2.08	2.12	0.49	+0.01	-0.06	+0.02	.....
	1	4.37	4.44	0.01	- .02	.00	- .07	0.00
	1r	5.09	5.26	1.57	.14	- .19	+ .02	+ .07
	2	5.28	5.38	-0.11	.08	+ .01	- .11	- .07
	3	5.56	5.62	0.18	.04	- .02	- .04	.00
II.....	4	5.84	5.86	0.14	.01	.02	.00	+ .02
	2s	6.30	6.28	0.17	.00	.02	+ .04	+ .04
	3s	6.35	6.33	0.26	.00	.03	+ .05	+ .04
	2r	6.32	6.55	1.58	.27	- .20	- .03	- .05
	5	6.45	6.43	0.01	.01	.00	+ .02	.00
III.....	6	7.06	7.01	0.06	- .02	- .01	.06	.00
	7	7.55	7.44	-0.17	+ .01	+ .02	.09	- .01
	3r	7.57	7.57	1.41	- .12	- .18	.18	+ .07
	8	8.13	7.97	0.25	+ .01	.03	.19	+ .05
	4r	8.27	8.26	1.00	- .16	.14	.15	- .01
IV.....	5r	8.63	8.67	1.53	- .24	.22	.18	- .02
	9	8.83	8.61	0.17	+ .03	.02	.24	+ .04
	10	9.06	8.94	0.12	- .09	.02	.14	- .09
	6r	9.24	9.25	1.26	- .23	.19	.18	- .07
V.....	11	9.56	9.30	0.24	+ .03	.04	.30	+ .04
	12	9.77	9.54	0.31	.00	.05	.28	.00
	4s	9.83	9.58	0.50	+ .02	.08	.33	+ .05
	7r	9.87	9.84	1.11	- .19	.19	.22	- .05
	5s	10.06	9.86	1.06	.02	.18	.38	+ .11
VI.....	13	10.37	10.25	0.22	.08	.04	.16	- .07
	8r	10.46	10.39	0.98	.11	.17	.24	.00
	14	10.56	10.44	0.42	.06	.08	.20	- .01
	6s	10.72	10.75	0.68	.18	.13	.10	- .08
	15	10.88	10.78	0.38	- .05	.07	.17	- .01
VII.....	16	11.22	11.07	0.36	+ .02	.07	.22	+ .07
	17	11.30	11.18	0.54	+ .01	.11	.22	+ .08
	18	11.90	11.93	0.39	- .08	.08	.05	- .03
	11r	12.07	12.24	1.12	.20	.27	.00	- .04
	19	12.24	12.22	0.44	.01	.11	+ .13	+ .09
	20	12.52	12.65	0.50	.09	- .12	- .01	-0.01
	23	13.00	13.34	0.56	.24	- .16	- .18	.....
	9s	13.74	13.94	1.04	-0.03	-0.31	+0.11	.....

imum difference is direct observation *minusc* alculat<sub>ion</sub> = +0.08 mag., at  $m=8.6$ . (4) Unpublished photovisual data obtained at the Harvard Observatory and communicated some years ago by Miss Leavitt agree well, in general, with the Mount Wilson results. The comparison extends from 6.5 to 14.5. A small divergence enters at the eighth magnitude; at the tenth magnitude it attains a maximum of +0.10 mag., in the sense  $MW-H$ , and thereafter remains nearly constant.

The Mount Wilson photovisual magnitudes of the *Report* are accordingly adopted as provisional standards, and for brevity the scale defined by them is frequently referred to in what follows simply as the "photovisual scale."

## 2. HARVARD VISUAL MAGNITUDES OF THE POLAR SEQUENCE

The differences in column 16, Table I, of the *Report* show that these magnitudes differ systematically from the photovisual scale. A color equation of approximately  $-0.14 C$  is also indicated.<sup>1</sup> A detailed examination of these differences follows.

The numbers, the Mount Wilson photovisual, and the Harvard visual magnitudes of the thirty-seven stars available for discussion are in the second, third, and fourth columns of Table I. The color-indices in the fifth column are the means of those found from magnitudes and by the method of exposure ratios, and given in the *Report* under the designation  $C_m$  and  $C_e$ , respectively.

The differences  $Pv-HV$  were provisionally corrected for color by applying  $-0.14 C$ . This led to an approximate scale correction which was then applied to the original values of  $Pv-HV$ , the results being in the sixth column of Table I. For the final determination of the color equation these corrected differences were grouped as indicated in the first column, the first star and the last three stars being omitted because of uncertainty in the scales for the extremes of brightness.

Each of the seven groups includes one or two red stars and several having small or moderate color-indices. Graphical solu-

<sup>1</sup> The sign, here as elsewhere in this discussion, is such that the algebraic addition of the correction reduces the data in question to the adopted standard system. Unless otherwise stated, the color-index  $C$  is that referred to the standard system.



tions for the separate groups gave the following values of the color coefficient:

Group.....	I	II	III	IV	V	VI	VII	} (1)
Mean vis. mag.	5.26	6.55	7.90	8.96	9.85	10.57	12.24	
Color coeff.....	-0.08	-0.18	-0.14	-0.15	-0.18	-0.17	-0.24	

These results indicate an increase in color equation with increasing magnitude. The smoothed values (that for  $m=3$  is from section 3) are

COLOR EQUATION, HARVARD VISUAL MAGNITUDES

Vis. mag.....	3	5	6	7	8	} (2)
Color coeff.....	-0.09	-0.11	-0.12	-0.13	-0.14	
Vis. mag.....		9	10	11	12	
Color coeff.....		-0.15	-0.17	-0.20	-0.23	

These were used to correct once more the original differences  $P_v-HV$ , the corrections themselves being in the seventh column

TABLE II

NORTH POLAR SEQUENCE, SCALE DIFFERENCE  $P_v-HV^*$

$HV$	$P_v-HV$	$HV$	$P_v-HV$	$HV$	$P_v-HV$
4.0.....	-0.08	7.5.....	+0.10	11.0.....	+0.16
4.5.....	.07	8.0.....	.14	11.5.....	.11
5.0.....	.06	8.5.....	.19	12.0.....	.06
5.5.....	.04	9.0.....	.24	12.5.....	+ .02
6.0.....	- .01	9.5.....	.28	13.0.....	- .03
6.5.....	+ .02	10.0.....	.25	13.5.....	-0.07
7.0.....	+0.06	10.5.....	+0.20	.....	.....

\* The correction to the magnitudes of the *Revised Harvard Photometry* for  $m < 4.0$  is -0.08. This, together with the tabular values, represents the provisionally adopted scale correction for reduction of the Harvard visual to the Mount Wilson photovisual scale. The corrections have been freed from color equation by means of the data under (2).

of Table I. The results in the eighth column define the adopted correction to the Harvard visual scale of the Polar Sequence. A plot of the differences is shown in Figure 1; the smoothed values are given in Table II, and, finally, the last column of Table I shows the residual differences  $P_v-HV$  found with the aid of (2) and the smoothed scale correction in Table II.

The corrected difference for Polaris in the first line of Table I is +0.02, which suggests that for the interval  $m=2$  to 6 the scales are



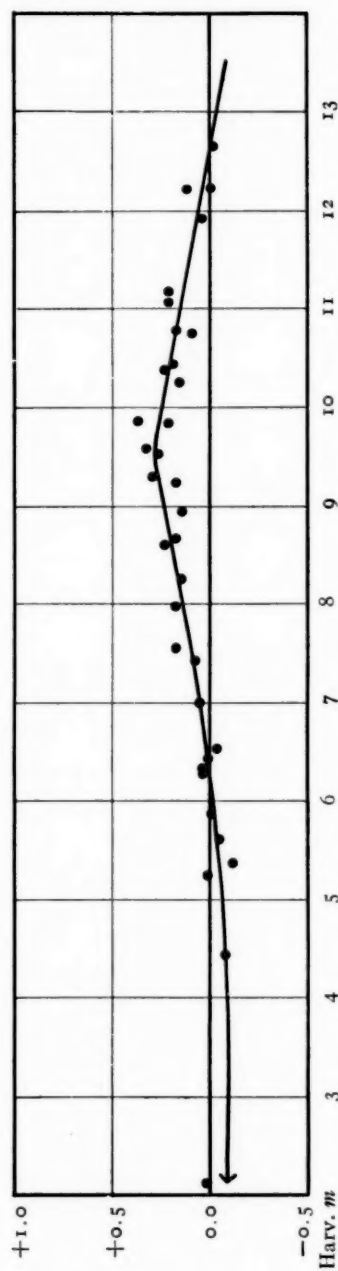


FIG. 1.—Reduction of the Harvard visual scale to the Mount Wilson photovisual system. The ordinates represent the differences  $P_V - HV$ , reduced to the photovisual color system.

nearly parallel. Between  $m=6$  and  $m=9.5$  the divergence is approximately represented by  $+0.07(m-6)$ , while below 9.5 it has the form  $-0.08(m-12.5)$ . This peculiar behavior probably represents the characteristics of the instruments commonly used for the Harvard visual measures—the 2-inch, 4-inch, and 12-inch meridian photometers. Generally speaking, measures with the 2-inch photometer extend to the sixth magnitude, and with the 4-inch to the tenth magnitude. With allowance for overlapping, these limits agree with the division-points of the run in the differences between the two scales.

### 3. COMPARISON OF THE HARVARD VISUAL SCALE WITH THE EXTRA-FOCAL MEASURES OF KING

The visual scale for the bright stars—practically that of the 2-inch meridian photometer—can be checked by comparison with the photographic and photovisual magnitudes of King.<sup>1</sup> The conclusions assume the substantial accuracy of King's scales, but there is reason to believe that this assumption is justified. It is based partly on the character of the method and the care with which it was applied, and partly on the good agreement of King's photographic magnitudes of the Polar Sequence with the international photographic scale.<sup>2</sup>

Consider first the color relations. The color-indices based on King's photographic magnitudes ( $KPg$ ) and the visual magnitudes of the *Revised Harvard Photometry* ( $HR$ ) which are given in *Harvard Annals*, 76, No. 6, Table IX, appear in the second column of Table III below.<sup>3</sup> The third column of the table gives the color-indices derived from his extra-focal photographic and photovisual measures, taken from *ibid.*, 85, No. 3, Table VI. The differences between the two series,  $KPv-HR$ , are in the fourth column of Table III. Similar differences found by comparing the photovisual magnitudes directly with the  $HR$  magnitudes and thus based on somewhat different data are in the fifth column. These are from Table V (*ibid.*),

<sup>1</sup> *Harvard Annals*, 76, No. 6; 85, No. 3.

<sup>2</sup> These magnitudes constitute Group 1 in Miss Leavitt's memoir in *Harvard Annals*, 71, No. 3.

<sup>3</sup> These are also given in *ibid.*, 85, No. 3, Table VI.

after correction by  $-0.13$  mag. to make the differences for A<sub>0</sub> stars equal to zero. The means for the fourth and fifth columns of Table III are in the sixth column, and define the color equation of the Harvard visual magnitudes of bright stars relative to King's photovisual system. These mean differences are represented by

$$KP_v - HR = -0.05 - 0.09 C. \quad (3)$$

This relation is based on stars brighter than  $m = 5$ , and refers to

TABLE III  
COMPARISON OF COLOR-INDICES

Harv. Sp.	$KPg - HR$	King $Pg - Pv$	$KP_v - HR$	$P_v - Plm$	$P_v - Vis$	MWC $M=0$	KC Smoothed	MWC $-KC$
B <sub>0</sub> .....	-0.29	-0.23	-0.06	-0.05	-0.06	-0.32	-0.23	-0.09
B <sub>1</sub> .....	.28	.17	.11	+ .06	.03	.29	.22	.07
B <sub>2</sub> .....	.34	.26	.08	- .08	.08	.26	.20	.06
B <sub>3</sub> .....	.25	.20	.05	+ .02	.02	.23	.19	.04
B <sub>5</sub> .....	.17	.10	.07	- .01	.04	.17	.15	- .02
B <sub>8</sub> .....	- .12	.08	- .04	.07	- .05	- .07	- .08	+ .01
A <sub>0</sub> .....	.00	- .02	+ .02	.00	+ .01	.00	.00	.00
A <sub>2</sub> .....	+ .12	+ .13	- .01	.06	- .03	+ .07	+ .06	.01
A <sub>3</sub> .....	.20	.18	+ .02	.05	.01	.11	.10	.01
A <sub>5</sub> .....	.20	.21	- .01	- .04	- .02	.19	.17	.02
F <sub>0</sub> .....	.33	.25	+ .08	+ .03	+ .06	.38	.35	.03
F <sub>5</sub> .....	.45	.59	- .14	- .09	- .12	.62	.59	+ .03
F <sub>8</sub> .....	.56	.80	.24	.14	.19	.77	.78	- .01
G <sub>0</sub> .....	.70	0.88	.18	.15	.17	0.86	0.85	+ .01
G <sub>5</sub> .....	0.98	1.16	.18	.21	.19	1.15	1.13	.02
K <sub>0</sub> .....	1.12	1.28	.16	.16	.16	1.48	1.43	.05
K <sub>2</sub> .....	1.50	1.61	.11	.18	.14	1.63	1.59	.04
K <sub>5</sub> .....	1.62	1.73	.11	.16	.13	1.84	1.73	.11
M.....	+1.66	+1.87	-0.21	-0.19	-0.20	+1.88	+1.87	+0.01

a mean magnitude of approximately 3. Since the various spectral groups in Table III include stars of all magnitudes within the interval covered, (3) may be regarded as unaffected by differential errors in the magnitude scales.

It is probably safe to assume that the color equation of King's photovisual system relative to the Mount Wilson system is zero. The similarity in color value of photovisual magnitudes derived with different instruments and plates has been mentioned. In the present case, the same kind of plate and the same type of filter were employed at both observatories. The only difference is in the telescopes—the 8- and 11-inch Draper, and the 13-inch Boyden

refractors, as against the 60-inch reflector. Since the instruments used by King are intermediate in color sensibility to the reflector and the Mount Wilson Cooke refractor, which give identical photovisual results, it is unlikely that his measures will show any systematic deviation in color from the Mount Wilson series.

The matter can be tested by comparing King's color-indices in the third column of Table III (the smoothed values in the eighth column are used) with the Mount Wilson indices in the seventh column, for which, since the stars are bright, values corresponding to absolute magnitude zero are used. The differences in the two series given in the last column of the table are represented by

$$\text{Color-index, } MW - \text{King} = +0.05 C. \quad (4)$$

If it be assumed, as suggested above, that the two photovisual systems are identical in color value, relation (4) will then represent the color equation of King's photographic system relative to the Mount Wilson system. As a matter of fact, the coefficient of (4) is very nearly that to be expected. In discussing<sup>1</sup> the photographic magnitudes in Miss Leavitt's memoir (*H.A.*, 71, No. 3), the following color equations were found

$$\left. \begin{aligned} MW - I &= +0.06 C \text{ (8-inch Draper telescope)} \\ MW - C &= +0.08 C \text{ (11-inch Draper telescope)} \end{aligned} \right\} \quad (5)$$

Further, King (*ibid.*, 76, 116) finds that the color sensibility of all three of the telescopes used by him is practically the same. A mean coefficient of +0.07 is therefore probably near the truth, and this, within the limit of uncertainty, agrees with +0.05 given in (4).

This seems to justify the conclusion that the color values of the two photovisual systems are the same; but it should be remarked that King's measures are extra-focal, while the results under (5) depend on focal images. The color value of such images may depend to some extent upon the aberrations of the objective, but the glass absorption is probably the more important factor.

In spite of this uncertainty, the general argument as to the agreement of the photovisual system holds, so that the term  $-0.09 C$  in (3) is probably a good approximation for the color equa-

<sup>1</sup> *Mt. Wilson Contr.*, No. 98; *Astrophysical Journal*, 41, 259, 1915.

tion of the Harvard visual measures of the bright stars. This result (for  $m=3$ ) is in good agreement with what might be expected from an extrapolation of the smoothed values under (2).

The relation between the magnitude scales can now be considered. This is best done by transforming King's photographic magnitudes into photovisual values with the aid of the color-indices for the various spectral types and comparing them with the *HR* magnitudes corrected for color equation.

The photographic magnitudes were taken from King's list in *Harvard Annals*, 76, 117, Table VIII, which contains 229 stars. Doubles and a few O stars, for which no color-indices are available, were rejected, thus leaving a total of 198 stars suitable for discussion. The difference,  $KPg - HR$ , was formed for each star, from which was derived  $KPg - RHR$ , where  $RHR$  indicates the *HR* magnitude reduced to the photovisual system by applying the appropriate correction from the sixth column of Table III. The remaining step involves the subtraction of the color-index  $KPg - KPv$  taken from the third column of this table. The result is  $KPv' - RHR$ .<sup>1</sup> No smoothing was employed, and the corrections from Table III were used as they stand, except that allowance was made for the dependence of color-index upon absolute magnitude. This variation in color is clearly shown by the differences  $KPg - RHR$  for the late F's and the G and K stars, in agreement with earlier results for these classes.<sup>2</sup> The changes range from  $-0.025 M$  for F8 to  $-0.044 M$  for types G5-K0.

Eight of the 198 corrected differences  $KPv' - RHR$  exceed 0.3 mag.; seven of these are positive, which suggests, for some at least, an excess of color over the normal for the spectral type in question. Four differences exceeding 0.4 mag. are clearly abnormal and were rejected. The remaining values were grouped according to the *HR* magnitude and formed into means, with the results shown in Table IV. These represent the systematic difference between the

<sup>1</sup> The notation  $Pv'$  denotes photovisual magnitudes calculated from photographic values, as distinguished from those directly measured.

<sup>2</sup> Seares, *Mt. Wilson Comm.*, No. 59; *Proc. Nat. Acad. of Sciences*, 5, 232, 1919. The variation for the A stars given in this *Communication* is probably illusory, and to be explained by a systematic error in the spectral classification whose existence was then unknown. The behavior of the B stars is still in doubt.

scale of the brighter stars of the *Revised Harvard Photometry* (essentially that of the 2-inch meridian photometer) and King's photographic scale after correction for color.

To about the fourth magnitude the scales are very accordant. Since 20 of the 25 differences included in the last two means are positive, the small systematic divergence at the end is probably real, although it is difficult to say which of the two scales is in error. The mean of the 194 differences without regard to sign is  $\pm 0.085$  mag. This represents the mean difference between the  $Pv'$  and  $HR$  magnitudes of the same star, including all sources of error, and indicates a probable error of about  $\pm 0.05$  mag. in the magnitudes. The zero-point of King's magnitude depends upon an assumed photographic magnitude of 2.69 for Polaris.

TABLE IV

SCALE DIFFERENCE—KING PHOTOVISUAL (CALCULATED FROM  $KPg$ ) AND *Revised Harvard Photometry*, REDUCED TO PHOTOVISUAL COLOR SYSTEM\*

$HR$ Mag.	$KPv' - RHR$	No. Stars	$HR$ Mag.	$KPv' - RHR$	No. Stars
0.0-0.5	+0.006	5	3.0-3.5	-0.006	42
0.5-1.0	-0.070	4	3.5-4.0	-0.004	22
1.0-1.5	+0.011	7	4.0-4.5	+0.086	19
1.5-2.0	+0.032	16	4.5-5.0	+0.077	6
2.0-2.5	-0.033	28			
2.5-3.0	+0.002	45	Total		194

\* Zero-point for King corresponds to 2.69 as photographic magnitude for Polaris.

There remains the comparison of King's photovisual magnitudes with those of the  $HR$ —more direct, but less satisfactory than that just given, because of the smaller number of stars and the shorter interval of brightness covered. The eighth column of Table IV, *Harvard Annals*, 85, No. 3, gives directly the differences  $Pv - HR$ . Rejecting doubles, correcting for the color equation of  $HR$  as before, and grouping the differences according to magnitude, we find the means given in Table V. The differences show a slight tendency to increase with magnitude, but in the well-determined region the scales are practically parallel.

The mean zero-point difference of  $+0.12$  mag. depends upon an assumed visual magnitude of 2.12 for Polaris. The difference found by King, without correcting the  $HR$  for color equation, is  $+0.13$ . To bring the zero-point of his  $Pv$  magnitudes into agreement with



that of the *HR*, and thus obtain his "completed" magnitudes, he subtracts this difference and finds accordingly for Polaris a photovisual magnitude of 1.99 and a color-index of 0.70. Part of this difference is accounted for by color equation. With allowance for this, the *HR* value for Polaris becomes 2.07. The Mount Wilson value is 2.08, so that the agreement seems to be very good. But another point remains still to be considered.

The international zero-point for the stars at the Pole was determined by the condition that the mean magnitude of the *Ao* stars between  $m = 5.5$  and 6.5 should equal the Harvard value; but owing to the small number of stars this condition may not have been exactly satisfied. Here we are adopting the polar stars as standards, and

TABLE V  
SCALE DIFFERENCE—KING PHOTOVISUAL AND *Revised Harvard Photometry*,  
REDUCED TO PHOTOVISUAL COLOR SYSTEM\*

<i>HR</i> Mag.	<i>KPv-RHR</i>	No.	<i>HR</i> Mag.	<i>KPv-RHR</i>	No.
0.0-0.5.....	+0.07	4	2.5-3.0.....	+0.11	35
0.5-1.0.....	.09	4	3.0-3.5.....	.19	12
1.0-1.5.....	.12	4	3.5-4.0.....	+0.13	1
1.5-2.0.....	.14	13			
2.0-2.5.....	+0.11	20	Total.....		93

\* Zero-point for King corresponds to 2.12 as photographic magnitude for Polaris. To obtain agreement with *HR*, King has subtracted 0.13 from the magnitudes used here. His "completed" magnitudes are the differences thus found.

attempting to find the relation of the Harvard magnitudes to this system. The evidence indicates that for  $m < 4$  the correction to the *HR* magnitudes is a constant. If we were to accept the Mount Wilson *Pv* magnitude of Polaris, 2.08, as exact, King's photovisual measures, all of which are referred to Polaris, would indicate that the *HR* requires (for the bright stars) a zero-point correction of +0.08 mag. to reduce it to the photovisual system. On the other hand, King's photographic measures combined with the international photographic magnitude of Polaris (2.55 as against 2.69 used by King) would lead to a zero-point correction of -0.14 mag. for the *HR*. These more or less hypothetical corrections include the full amount of the accidental errors in the magnitudes adopted for Polaris and any systematic divergence in King's scales. The internal evidence and the small run in the differences in Table V

suggest that King's measured differences in photovisual magnitudes are slightly too large. A reasonable allowance would reduce the zero-point correction of  $+0.08$  mag. to zero.

If it were certain that King's photographic scale is accurate at the lower end, the corrections to the *HR* magnitudes of the bright stars could be found by using the condition that the corrected values of the last two differences in Table IV must agree with the initial values in Table II. The correction thus indicated would be  $-0.16$  mag., which agrees well with  $-0.14$  mag., based on King's photographic measures and the international photographic magnitude of Polaris. But as shown in section 5, King's photographic magnitudes for the fainter stars observed by him deviate from the *PD* values in the same way that they do from the *HR*. The positive differences at the end of the series in Table IV perhaps indicate, therefore, a small divergence in his scale for  $m < 4$ .

It is probably safer, therefore, to accept the mean of the corrections indicated by the magnitudes of Polaris, namely,  $-0.14$  and  $0.00$  mag., respectively. Although uncertain, a correction of  $-0.07$  mag. for  $m < 4.0$  is consistent with results in Table II and is satisfactorily confirmed by the more reliable discussion in section 6.

#### 4. COMPARISON OF THE HARVARD VISUAL SCALE WITH THAT OF THE *Potsdam Durchmusterung*

The complicated character of the relation between these scales is well known from the comparisons of Pickering,<sup>1</sup> Müller and Kempf,<sup>2</sup> and Schwarzschild.<sup>3</sup> Consider first the comparison of Müller and Kempf, who have formed the mean differences for four color classes and each half-magnitude interval between the *Potsdam Durchmusterung* and *Harvard Annals*, 44 and 45. The latter catalogues deviate slightly from the *Revised Harvard Photometry*, but are easily reduced to the *HR* system by the corrections given in *Harvard Annals*, 64, 117, 118, supplemented by the following color equivalents:

Potsdam color class . . . . .	W	GW	WG	G	} (6)
Mt. Wilson color-index . . . . .	$+0.02$	$+0.30$	$+1.04$	$+1.66$	

<sup>1</sup> *Harvard Annals*, 64, 119.

<sup>2</sup> *Pub. des Astrophys. Observatoriums zu Potsdam*, 17, xxxiii, 1907.

<sup>3</sup> *Göttingen Aktinometrie*, B, 23, Table 20.



Harvard spectral group.....	B	A	F	G	K	M	(7)
Mt. Wilson color-index.....	-0.26	0.00	+0.38	+0.83	+1.48	+1.88	

Those under (6) were found from the data in Table 25 of the *Göttingen Aktinometrie*, B, 29, combined with the following relation between the Mount Wilson and Göttingen color-indices:

$$C = J_p + 0.29. \quad (8)$$

The values in (7) are the Mount Wilson colors for giant stars corresponding to the median spectral types of the various groups.<sup>1</sup>

TABLE VI  
MEAN DIFFERENCES,  $PD-HR$

MED. $HR$ MAG.	POTSDAM COLOR CLASS				REL. COL. EQ. $C$	COLOR EQ.		$PD-HR$ CORR. FOR $C$	$P_V-PD$	$P_V-HR$	$P_V-HR$ TABLE II	DIFF.
	$W$	$GW$	$WG$	$G$		$HR$ $C$	$PD$ $C$					
2.25.....	+0.27	+0.24	+0.15	+0.09	+0.10	-0.08	+0.02	+0.26	-0.26	0.00	.....	.....
2.75.....	.26	.23	.14	.08	.10	.00	.01	.25	.26	-.01	.....	.....
3.25.....	.25	.22	.12	.07	.11	.00	.02	.24	.26	.02	.....	.....
3.75.....	.24	.21	.11	.06	.11	.10	.01	.24	.26	.02	-0.08	-0.06
4.25.....	.24	.20	.10	.04	.12	.10	.02	.24	.26	.02	.08	.06
4.75.....	.25	.20	.09	.02	.14	.10	.04	.24	.26	-.02	.06	.04
5.25.....	.26	.21	.08	+.01	.16	.11	.05	.26	.25	+.01	.05	.06
5.75.....	.28	.23	.08	-.02	.18	.12	.06	.28	.23	.05	-.03	.03
6.25.....	.30	.24	.06	.02	.20	.12	.08	.30	.22	.08	.00	.08
6.75.....	.32	.27	.09	.02	.21	.13	.08	.32	.20	.12	+.04	.08
7.25.....	.33	.27	.08	.04	.22	.13	.09	.33	.19	.14	.08	.06
7.75.....	.34	.28	.08	.05	.24	.14	.10	.34	.17	.17	.12	.05
8.25.....	+0.35	+0.28	+0.07	-0.06	+0.25	-0.14	+0.11	+0.35	-0.16	+0.19	+0.16	-0.03
$C$ .....	0.02	0.30	1.04	1.66	.....	.....	.....	.....	.....	.....	Mean	-0.06

The reduction to the  $HR$  system thus outlined leads finally to the mean differences given in the first five columns of Table VI. Within the magnitude interval for which a comparison is possible, the results are in almost exact agreement with those found by Pickering. The differences include both relative scale error and color equation. The coefficients for the color equations appear in the sixth column of the table. These were found graphically by plotting the differences in each line of the four columns immediately preceding against the values of  $C$  given at the bottom of the table.

The color equation of the  $HR$  relative to the photovisual system

<sup>1</sup> *Mt. Wilson Contr.*, No. 287, Table I; *Astrophysical Journal*, 61, 114, 1925.

has already been found, and is given under (2). The values corresponding to the magnitudes listed in the first column of Table VI appear in the seventh column. Combining these with the data in the sixth column, we find the values of the color equation of the *PD* referred to the photovisual system given in the eighth column of Table VI. For the bright stars the correction is sensibly zero and increases slowly to about  $+0.10 C$  at  $m=8$ . The *HR* and *PD* systems therefore deviate from the photovisual systems in opposite directions, and the combination of the two equations, differing in sign, is responsible for the large relative equation shown in the sixth column of the table.

The differences in the second to the fifth columns of Table VI, corrected for relative color equation as given in the sixth column, give the mean differences in the ninth column. These represent the scale divergence of the two systems in the sense  $PD-HR$ . The scales are sensibly parallel to about  $m=5$ ; thereafter the difference is represented by

$$PD-HR = +0.25 + 0.03 (HR-5.0). \quad (9)$$

The remaining columns of Table VI are considered in section 6.

A characteristic feature of the comparisons between the *HR* and *PD* systems made by Pickering and by Müller and Kempf is the dependence of color equation upon magnitude. In one case the magnitudes were grouped according to Harvard spectra taken from the original *Draper Catalogue*, in the other, according to Potsdam color estimates. As already mentioned, the results are sensibly identical. In contrast to these are the two comparisons by Schwarzschild,<sup>1</sup> one, based on a grouping according to the spectral types of Miss Cannon in *Harvard Annals*, 56, No. 4, the other depending on the Göttingen color-indices. His formulae are, respectively:

$$\begin{aligned} PD-HR &= +0.45 + 0.046 (HR-6.5) - 0.14 Sp \quad (\text{Sch. 5}) \\ PD-HR &= +0.52 + 0.086 (HR-6.5) - 0.13 Sp \quad (\text{Sch. 11}) \end{aligned}$$

where *Sp* indicates the spectral type, as defined on page 19 of the *Aktinometrie*.

Here, as Schwarzschild notes, the color terms do not involve the

<sup>1</sup> *Göttingen Aktinometrie*, B, eq. 5, p. 26, and eq. 11, p. 25.

magnitude. His suggested explanation is that the *Draper Catalogue* spectra and the Potsdam color estimates are affected by systematic errors. Whether this is the case or not would be immaterial were the various formulae always used in connection with the data upon which they are based, but it does raise some question as to the reduction in Table VI, for which color-indices have been used on the assumption that the same value of the index always corresponds to a given color estimate.

For comparison with the earlier results of this section, Schwarzschild's formulae may be expressed in terms of the standard color-index with the aid of equation (4) of the *Aktinometrie*,

$$J_p = G - P = -1.03 - 0.09 (PD - 6.5) + 0.63 Sp \quad (10)$$

combined with (8) above, from which

$$Sp = 1.59 C + 1.18 + 0.143 (PD - 6.5). \quad (11)$$

This formula does not refer to the general catalogue of the *PD*, but to one of the earlier volumes; and the final expression for *Sp* in terms of *C* requires the substitution of equation (9) of the *Aktinometrie*. This gives

$$Sp = 1.59 C + 0.140 PD + 0.264, \quad (12)$$

whence Schwarzschild's comparison formulae become

$$PD - HR = +0.27 + 0.025 (HR - 6.5) - 0.22 C \quad (13)$$

$$PD - HR = +0.36 + 0.067 (HR - 6.5) - 0.20 C. \quad (14)$$

The magnitude intervals covered are about 4.5 to 7.5 and 5.0 to 8+, respectively, while the mean magnitudes are 5.6 and 7.0. The relative color equations corresponding to these means, given in the sixth column of Table VI, are 0.18 *C* and 0.21 *C*, respectively, which are in satisfactory agreement with the color terms in (13) and (14).

The scale differences expressed by the first two terms of (13) and (14) are discordant, but that corresponding to (13), which is directly comparable with (9), reproduces the differences in the ninth column of Table VI, for the interval in question, within the uncertainty of the comparison. The outstanding difference is in the zero-point and amounts to only 0.03 mag. As far as scale difference is concerned,

Schwarzschild's comparison based on Miss Cannon's spectra in *Harvard Annals*, 56, No. 4, is therefore in agreement with the results of Pickering and Müller and Kempf. The larger divergence which appears when the data are grouped according to Göttingen color-indices and the apparent constancy of the color equation shown by both Schwarzschild's comparisons remain obscure.

#### 5. COMPARISON OF KING'S EXTRA-FOCAL MEASURES WITH THE *Potsdam Durchmusterung*

Some further light is thrown on the relation of scales and color equations by comparing King's magnitudes with the visual magnitudes of the *PD*. Because of their greater number, the photographic series was used, after reduction to the photovisual system by the color-indices in the third column of Table III. The differences  $KP_v' - PD$  for the 115 stars common to the two lists were grouped according to spectral type and plotted. The stars are all brighter than the fifth magnitude, and most of them fall between  $m = 2.0$  and 4.5. Within this interval the differences are well represented by

$$KP_v' - PD = -0.14 + 0.04 (PD - 6). \quad (15)$$

The systematic deviations from this formula for the different spectral types are only 0.01 or 0.02 mag. To  $m = 4.5$  or 5.0, therefore, the color equation of the *PD* magnitudes relative to the photovisual system is sensibly zero. This confirms the very small values of the color coefficient for the *PD* system shown in the upper part of the eighth column of Table VI.

As for scale difference, the coefficient of 0.04 in (15) does not accurately represent the relations. If the stars fainter than  $m = 3.0$  be disregarded, the differences can be represented satisfactorily by a constant. Except for a zero-point correction, the scale divergence is therefore very similar to that shown by the differences  $KP_v' - RHR$  in Table IV. And this, indeed, is to be expected, for the mean differences given in the ninth column of Table VI show that to the fifth magnitude the Harvard and Potsdam scales are sensibly parallel. This suggests that the small scale-divergence shown by the last two differences in Table IV is in King's scale rather than in that of the *HR*.

6. COMPARISON OF THE POTSDAM MAGNITUDES OF THE  
POLAR SEQUENCE STARS WITH THE MOUNT WILSON  
PHOTOVISUAL VALUES

The number of stars available for this comparison is only twelve, but their magnitudes are well determined and cover the critical interval from  $m=4.7$  to  $7.7$  in such a way as to give, in connection with the  $PD$  and  $HR$  comparison, a fairly reliable indication as to the characteristics of the Harvard scale in this region. Because of the importance of the comparison, the details are shown (Table VII).

TABLE VII  
POLAR SEQUENCE STARS—POTSDAM AND MOUNT WILSON

Star	$MW P_v$	$C$	$PD Vis.$	$P_v - PD$	$O - C$
15.....	2.08	+0.49	2.31	-0.23	0.00
1.....	4.37	0.01	4.69	.32	- .06
17.....	5.09	+1.57	5.18	.09	+ .05
2.....	5.28	-0.11	5.37	.09	(+.17)
3.....	5.56	+0.18	5.82	.26	- .03
4.....	5.84	0.14	6.10	.26	- .04
25.....	6.30	0.18	6.50	.20	+ .01
27.....	6.32	1.58	6.47	.15	- .04
35.....	6.35	0.26	6.48	.13	+ .07
5.....	6.45	0.02	6.70	.25	- .04
6.....	7.06	+0.06	7.28	.22	- .03
7.....	7.55	-0.16	7.70	-0.15	+0.04

The uncorrected differences in the fifth column are represented by the formula

$$P_v - PD = -0.26 + 0.03 (PD - 5.0) + 0.07 C, \quad (16)$$

in which the second term is to be disregarded for stars brighter than 5.0. The residuals in the last column of Table VII show only one discordant value. The mean magnitude of the stars is about six. The color equation of  $+0.07 C$  is therefore in agreement with the results given in the eighth column of Table VI.

The ninth column of Table VI gives the values of  $PD - HR$ , reduced to the photovisual system of color. If now the  $PD$  scale correction defined by the first two terms of (16) be applied to these differences, the result will be the scale differences  $P_v - HR$ . These are given in the eleventh column of Table VI. Because of the indirect nature of the comparison and the small number of polar

stars, the zero-point may require correction; but the characteristics of the *HR* scale in the critical region between  $m=4$  and 7 should be reasonably well defined. For comparison, the results from Table II, found by a direct comparison of magnitudes of Polar Sequence stars, are given in the twelfth column of Table VI. The differences in the last column indicate a mean zero-point correction of  $-0.06$  mag. for stars involved in the comparison of Table VII; but what is important is the fact that these differences are sensibly constant. This shows that the relative values of the corrections to the *HR* scale found by the two methods are sensibly the same and that the initial values of the correction to the Harvard scale given in Table II are therefore probably near the truth. Further, since the *HR* and *Pv* scales for  $m < 4.0$  seem to be parallel, the constant correction required to reduce these stars to the photovisual system may be taken as  $-0.08$  mag. This agrees well with the rather uncertain result found in section 3, and is the value given in the note to Table II.

MOUNT WILSON OBSERVATORY  
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# REDUCTION OF THE HARVARD-GRONINGEN *DURCHMUSTERUNG* TO THE INTERNATIONAL SYSTEM OF MAGNITUDE AND COLOR<sup>1</sup>

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## ABSTRACT

*The data.*—The *Durchmusterung* in *HA*, 101, 102, 103, gives the brightness of stars in Selected Areas 1-206, to a limiting magnitude of about 16.0. Many of these appear in the Mount Wilson *Catalogue of Selected Areas*, which includes Areas 1-139. It is shown that the *Catalogue* is on the international photographic scale. The *Durchmusterung* can therefore be reduced to the latter scale by direct comparison of magnitudes for stars observed in common.

*Color equation and scale correction for bright stars.*—The *Durchmusterung* magnitudes for the northern hemisphere, measured with the Metcalf 16-inch telescope, are affected by a large color equation. This was determined from the central stars of the areas, whose color-indices are known. The complete reduction thus found from eighty stars for the Metcalf telescope before the objective was reground is

$$MW - DM = +0.06 (DM - 6) + 0.21 C \quad (m \geq 10). \quad (9)$$

The scale-divergence is the same as that of *HA*, 71, No. 3, as it should be.

*Scale and color correction for faint stars.*—Corrections for each area, for scale and color combined, were derived from the stars common to the two catalogues (Table III). With allowance for color equation based on the mean color-index of stars of different magnitudes, it is found that to  $m=13$  the scale correction of the *DM* is the same as that of *HA*, 71, No. 3 (Table IV), but for fainter stars there is an appreciable divergence. The fluctuations in the corrections from area to area are large, and obscure the systematic difference for stars of high and low galactic latitude, which depends on the fact that the former are redder than the latter. For statistical purposes the separate areas should be reduced to the international system with the aid of Table III.

The following discussion relates to the stars of the Harvard-Groningen *Durchmusterung of the Selected Areas*<sup>2</sup> included in the 139 regions on and north of declination  $-15^\circ$ . The fields measured were  $40' \times 40'$ ,  $60' \times 60'$ , or  $80' \times 80'$ , according to galactic latitude. The *Durchmusterung* includes the stars in these fields to a limiting photographic magnitude of about 16.0.

These data form an important contribution to the study of stellar distribution, but, as they stand, cannot be combined with other results, since they are referred to the scale of *Harvard Annals*, 71, No. 3, which deviates appreciably from the international scale.

<sup>1</sup> *Contributions from the Mount Wilson Observatory*, No. 289.

<sup>2</sup> *Harvard Annals*, 101, 1918; 102, 103, 1924.



Moreover, there is a difficulty in that about half of the photographs were made with the 16-inch Metcalf telescope, which is known to have a large color equation relative to most other instruments.<sup>1</sup> Referred to the international system,<sup>2</sup> the color equation is 20 or 25 per cent of the color-index; and since the mean color of the stars, grouped according to photographic magnitude for all galactic latitudes together, is represented by the linear relation<sup>3</sup>

$$C_p = -0.18 + 0.071m_p, \quad (1)$$

it follows that the effect of the color equation on star counts will be considerable and will enter as an additional disturbance of scale, which for the faint stars is of considerable importance. The remaining photographs for the *Durchmusterung* were made with the Bruce telescope whose color equation is still to be investigated.

The simplest method of determining the total systematic deviation from the international scale is to compare the magnitudes of the *Durchmusterung* with those of the Mount Wilson *Catalogue of the Selected Areas*.<sup>4</sup> The latter catalogue gives the photographic magnitudes, on the international scale, of all the stars in fields 23' in diameter, centered on the central stars of Areas 1-139.

For stars fainter than the tenth or eleventh magnitude, the data are sufficient for a comparison of scales in individual areas. For the brighter objects, including chiefly the central stars, only the mean scale-divergence can be determined. As a matter of fact, the discussion can be carried somewhat farther, for the visual magnitudes of the central stars were measured by Professor Pickering with the 4-inch and 12-inch meridian photometers. Combined with the photographic magnitudes, these results give the colors of the individual stars, and hence permit a good determination of the color equation of the Metcalf telescope. With the aid of (1), the expres-

<sup>1</sup> See Table II, *Mt. Wilson Contr.*, No. 98; *Astrophysical Journal*, **41**, 259, 1915. In this tabulation MC indicates the Metcalf telescope.

<sup>2</sup> As defined by the adopted photographic magnitudes of stars near the Pole given in *Trans. Internat. Astron. Union*, **1**, 69, 1922.

<sup>3</sup> *Mt. Wilson Contr.*, No. 287; *Astrophysical Journal*, **61**, 114, 1925.

<sup>4</sup> For an account of this catalogue, which is complete but not yet in print, see P. J. van Rhijn, *Third Report on the Progress of the Plan of Selected Areas*, Groningen, 1923; particularly, the Second Appendix, by F. H. Seares.



sion for mean color, it then becomes possible to separate the influence of color equation from the true scale-divergence and thus obtain a comparison of the scales of the *Durchmusterung* and *Harvard Annals*, 71, No. 3, which are supposed to be identical.

Because of the manner in which they were derived,<sup>1</sup> the magnitudes of the *Durchmusterung* are necessarily approximate; but, as far as probable error goes, they are of ample precision for statistical purposes. Nevertheless, their approximate character is disadvantageous for the determination of the color equation, and the uncertainty is increased by the fact that this calculation is based on stars brighter than the tenth magnitude, for which the photographic magnitudes, whether Harvard, Groningen, or Mount Wilson values, are much less reliable than those of the fainter stars. The large differences affecting many of the stars are therefore not unexpected.

#### I. AGREEMENT OF THE MOUNT WILSON *Catalogue of Selected Areas* WITH THE INTERNATIONAL SCALE

Since the reduction of the *Durchmusterung* to the international scale depends upon its relation to the Mount Wilson *Catalogue*, it is important to show that this catalogue is really in agreement with the international system. Such agreement is to be expected, since the scale for the *Catalogue* was established in each area independently by the methods used in deriving the Mount Wilson magnitudes at the Pole,<sup>2</sup> which are known to be on the international scale.<sup>3</sup> The matter can be tested by means of the polar comparison photographs which were used to determine the zero-points of the scales in the individual areas. Thirty-seven of the fields, uniformly distributed over the northern heavens, were directly connected with the Pole by duplicate plates. These have been used to transfer the scale of the polar stars to the areas themselves for comparison with the scales adopted for the *Catalogue*. The magnitude interval thus covered is less than that of the *Catalogue*, because the comparison exposures were short, but sufficient for the present purpose. The

<sup>1</sup> *Harvard Annals*, 101, ix, 1918.

<sup>2</sup> *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, 41, 206, 1915.

<sup>3</sup> *Trans. Internat. Astron. Union*, 1, 69, 1922.

mean result of this comparison for the thirty-seven fields is as follows:

Med. mag.....	10.5	11.5	12.5	13.5	14.5	15.5	16.5
<i>Int. Sc.-MW Cat.</i> +0.09	-0.05	-0.01	+0.01	0.00	0.00	+0.05	
No. diff.....	16	75	181	381	519	401	70

With the exception of the first difference, for which the images on the comparison photographs are too large for reliable measurement, the corrections to the *Catalogue* necessary for reduction to the international system are negligible. This applies to zero-point as well as to scale errors.

## 2. COLOR EQUATION AND SCALE ERROR FOR THE BRIGHT STARS OF THE *Durchmusterung*

Forty of the 139 areas were photographed with the Bruce telescope; sixteen others were photographed with the Metcalf instrument after the regrinding of the objective. For the central stars of two others (Nos. 42, 48) there are no Mount Wilson magnitudes, while for a third (No. 30), the central star was measured as a double at Mount Wilson and a single star at Groningen. This leaves a total of eighty regions for the determination of the color equation of the Metcalf telescope and the scale error for the bright stars. Central stars in 39 of the areas photographed with the Bruce telescope (that in No. 126 was not measured at Mount Wilson) are also available for the discussion of the color equation, but these lead to no definite result. The magnitudes of about half of these stars are Harvard standards, supplied by Professor Pickering apparently on the basis of measures with the Metcalf telescope, while the others must also be influenced by the color equation affecting the standards.

The data for the central stars observed with the Metcalf objective in its original state are listed in Table I, the first four columns of which give in succession the number of the area, the Mount Wilson photographic magnitude, and the Harvard photographic and visual magnitudes taken from *Harvard Annals*, 101, xlviii; 102, vi. The sixth column gives the value of the color-index  $C_1 = MWPg - HV$ , and the eighth column, the difference between

TABLE I  
CENTRAL STARS—METCALF TELESCOPE, ORIGINAL OBJECTIVE

S.A.	MWP <sub>g</sub>	HP <sub>g</sub>	HV	V <sub>g</sub>	C <sub>1</sub>	C	MWP <sub>g</sub> — HP <sub>g</sub>
1.....	7.23	7.11	7.01	7.07	0.22	0.19	+0.12
3.....	8.51	8.64	8.62	8.84	-0.11	-0.38	- .13
4.....	9.94	9.37	8.41	8.60	1.53	1.55	+ .57
5.....	9.35	9.16	8.60	8.82	0.75	0.61	.19
6.....	10.49	10.10	8.91	9.15	1.58	1.55	.39
8.....	8.84	8.18	8.11	8.27	0.73	0.66	.66
10.....	8.89	8.76	8.59	8.81	0.30	0.09	.13
11.....	8.30	7.93	7.14	7.21	1.16	1.26	.37
12.....	8.64	8.48	8.17	8.33	0.47	0.36	+ .16
13.....	7.83	8.05	7.17	7.25	0.66	0.67	- .22
14.....	10.50	10.14	9.22	9.48	1.28	1.18	+ .36
15.....	9.98	9.55	8.35	8.53	1.63	1.68	.43
16.....	9.16	8.69	7.92	8.06	1.24	1.28	.47
17.....	9.05	8.44	7.51	7.61	1.54	1.67	.61
20.....	8.97	8.63	8.66	8.89	0.31	0.09	.34
21.....	10.03	9.50	8.53	8.74	1.50	1.50	.53
23.....	8.83	7.86	7.52	7.62	1.31	1.40	.97
24.....	10.28	9.81	8.92	9.16	1.36	1.30	.47
25.....	9.29	9.17	8.97	9.22	0.32	0.08	.12
26.....	10.47	10.15	9.35	9.61	1.12	1.00	.32
27.....	10.40	9.82	9.00	9.25	1.40	1.33	.58
28.....	10.59	9.93	9.06	9.21	1.53	1.60	.66
29.....	8.78	8.60	8.28	8.46	0.50	0.37	.18
31.....	9.49	9.24	8.72	8.95	0.77	0.63	.25
32.....	9.71	9.35	8.89	9.13	0.82	0.67	+ .36
33.....	9.40	9.42	9.02	9.27	0.38	0.15	-0.02
34.....	9.63	8.37	7.86	7.99	1.77	1.90	+1.26
35.....	10.85	9.63	8.91	9.15	1.94	1.97	1.22
36.....	9.34	9.00	8.75	8.98	0.59	0.42	0.34
37.....	10.58	9.71	9.20	9.46	1.38	1.30	.87
38.....	9.71	9.30	8.24	8.41	1.47	1.51	.41
39.....	9.16	9.02	8.06	8.21	1.10	1.10	.14
40.....	9.84	9.53	8.66	8.88	1.18	1.11	.31
41.....	10.06	9.84	9.31	9.57	0.75	0.57	.22
43.....	10.10	9.75	8.25	8.42	1.85	1.95	.35
44.....	8.96	8.78	8.70	8.93	0.26	0.03	.18
45.....	9.00	8.86	8.18	8.36	0.82	0.74	.14
46.....	9.51	9.19	8.36	8.53	1.15	1.11	+ .32
49.....	9.90	9.93	8.71	8.94	1.19	1.11	- .03
50.....	8.55	8.09	8.04	8.19	0.51	0.42	+ .46
51.....	10.71	10.05	9.56	9.82	1.15	1.03	.66
52.....	8.53	8.35	8.08	8.24	0.45	0.34	.18
54.....	10.23	9.53	9.26	9.52	0.97	0.82	.70
55.....	10.08	9.47	8.68	8.91	1.40	1.36	.61
56.....	10.13	9.53	8.23	8.40	1.90	2.01	.60
57.....	9.01	8.83	8.53	8.73	0.48	0.32	.18
58.....	9.72	9.35	7.68	7.80	2.04	2.23	.37
59.....	9.42	9.34	9.01	9.26	0.41	0.19	.08
60.....	9.50	9.30	8.57	8.78	0.93	0.84	.20
61.....	9.56	9.06	8.94	9.19	0.62	0.43	0.50
62.....	8.36	7.20	7.05	7.12	1.31	1.44	1.16
63.....	9.61	8.90	9.14	9.40	0.47	0.24	+0.71

TABLE I—Continued

S.A.	MWP <sub>g</sub>	HP <sub>g</sub>	HV	V <sub>e</sub>	C <sub>1</sub>	C	MWP <sub>g</sub> — HP <sub>g</sub>
64.....	8.31	8.12	7.62	7.73	0.69	0.67	+0.19
65.....	9.45	9.04	8.88	9.12	0.57	0.38	.41
66.....	10.10	9.68	9.04	9.29	1.06	0.94	.42
67.....	9.67	8.98	8.20	8.37	1.47	1.51	.69
68.....	9.08	8.13	8.27	8.45	0.81	0.73	.95
69.....	8.14	7.97	7.39	7.49	0.75	0.75	.17
70.....	8.20	7.63	7.90	8.04	0.30	0.19	.57
71.....	9.74	9.36	8.65	8.87	1.09	1.01	.38
72.....	6.89	6.82	6.22	6.24	0.67	0.75	.07
73.....	8.34	7.94	8.14	8.30	0.20	0.04	.40
74.....	8.80	8.86	9.13	9.38	-0.24	-0.57	.03
75.....	9.62	9.37	8.48	8.68	1.14	1.09	.25
76.....	9.57	9.09	9.03	9.28	0.54	0.34	.48
77.....	8.66	8.57	8.10	8.36	0.47	0.35	.09
78.....	9.58	9.43	8.40	8.59	1.18	1.15	.15
79.....	9.60	9.09	8.37	8.56	1.23	1.21	.51
80.....	8.53	8.35	8.21	8.38	0.32	0.17	.18
81.....	10.29	9.81	9.14	9.40	1.15	1.03	.48
82.....	9.74	9.44	8.26	8.44	1.48	1.51	.30
83.....	9.25	9.21	8.58	8.80	0.67	0.52	.04
84.....	8.82	8.65	8.52	8.72	0.30	0.12	.17
85.....	9.37	8.82	8.28	8.46	1.09	1.06	.55
86.....	10.49	9.69	9.18	9.44	1.31	1.22	.80
87.....	9.66	9.01	8.05	8.20	1.61	1.69	.65
88.....	8.82	8.16	8.66	8.88	0.16	-0.07	.66
89.....	8.70	8.22	8.11	8.27	0.59	0.50	+ .48
90.....	9.04	9.12	9.03	8.27	0.01	-0.28	- .08
100.....	9.27	9.06	8.49	8.69	0.78	0.67	+0.21

the Mount Wilson and Harvard photographic magnitudes. These differences include the influence of scale error, color equation, and the large accidental errors to which reference has been made. The fifth and seventh columns require more detailed explanation.

A reference to the sixteenth column of Table I of the *Report of the Commission on Stellar Photometry*<sup>4</sup> shows the existence of an appreciable difference between the Harvard visual and Mount Wilson photovisual scales and color systems. The matter is discussed in detail in *Contribution No. 288*, and there can be little doubt that the Harvard magnitudes require corrections for scale and color of the order of those given in Table II and under (2) in that paper. The scale correction has its maximum value between the ninth and tenth magnitudes, in the region where measures made with the 12-inch meridian photometer overlap those made with the 4-inch

<sup>4</sup> *Ibid.*, p. 71, 1922.

photometer. The color equation seems to increase with the magnitude.

For the present purpose, however, which is that of freeing the color-indices of the central stars of the Selected Areas from systematic errors, a constant mean value of the color equation is ample, and the formula for reduction to the photovisual system may be written

$$Pv = HV + \text{Table II, Contr. 288} - 0.139 C, \quad (2)$$

in which  $C$  is the color-index on the international system.

It is not certain that (2) is strictly applicable to the Harvard magnitudes in Table I. Whether this is the case or not probably depends on the manner in which measures with the two meridian photometers have been combined, and possibly on other factors as well. On this account, a double reduction has been made, using both the corrected and uncorrected values of the Harvard visual magnitudes.

For the actual discussion it is the corrected color-indices and not the visual magnitudes themselves that are required. The index  $C_1$ , corresponding to the uncorrected magnitudes, has already been found. The values of  $C$ , based on the corrected visual magnitudes, were derived as follows: The values of  $HV$  corrected for scale error are given in the fifth column of Table I under the heading  $V_c$ .<sup>1</sup> Introducing  $V_c$  into (2), we have

$$Pv = V_c - 0.139 C. \quad (3)$$

Further, for data on the international system,

$$Pg - Pv = C, \quad (4)$$

whence

$$C = 1.16 (Pg - V_c). \quad (5)$$

The values of  $C$  in the seventh column of Table I were calculated with this formula, using for  $Pg$  the Mount Wilson value,  $MWPg$ , given in the second column of the table.

The relation between the differences  $MWPg - HPg = n$ , given in the last column of Table I, and the coefficients of the scale and

<sup>1</sup> As a matter of fact, the scale corrections actually used were provisional values, differing in some cases by 0.02 or 0.03 mag. from those in Table II of *Contribution* No. 287. The differences are of no consequence for the present discussion.

color corrections to the  $Pg$  scale of the *Durchmusterung* can be expressed in the form

$$x + ym + zC = n. \quad (6)$$

The three unknowns,  $x$ ,  $y$ ,  $z$ , cannot be determined simultaneously, however, because of an inherent uncertainty in the solution. A preliminary examination showed that the scale coefficient  $y$  is very nearly 0.06, in agreement with the known error of the scale of *Harvard Annals*, 71, No. 3. Since the range in  $m$  is not great, it is convenient to adopt this provisional value of  $y$  and reduce all the differences  $n$  to  $m=9$ . Equation 6 accordingly becomes

$$x + zC = n - 0.06(m - 9), \quad (7)$$

where  $x$  is now the difference between the two scales at the ninth magnitude, corrected for color equation.

For the application of (7) the data for the eighty stars were combined into twelve groups, arranged according to  $HPg$  and  $C$  (or  $C_1$ ). The corrections arising from the last term in (7) do not exceed 0.05 or 0.06 mag., so that any error in the assumed value of  $y$  is unimportant.

Least-squares solutions for  $x$  and  $z$  thus gave

<i>HV</i> Uncorrected	<i>HV</i> Corrected	
$x = +0.154$	$x = +0.199$	(8)
$z = +0.243$	$z = +0.211$	

These results are a sufficient indication of the influence on the reduction of the difference between the Harvard visual and the Mount Wilson photovisual scales. For the further calculation the corrected  $HV$  series is used as the more probable of the two.

To investigate the coefficient  $y$ , which measures the scale-divergence, the original differences in the last column of Table I were corrected for color equation by subtracting from each the quantity  $0.211C$ , and then grouped according to Harvard photographic magnitude. The means for four groups are shown in the first two and the last two lines of Table II (first and second columns). Seven discordant differences were excluded (Areas 8, 23, 62, 68, 70,

and 88) from the first group, and that for Area 63 from the second. For the first group in particular, the discordant differences are so large that there can be no question of their abnormality, which doubtless arises from the brightness of the stars. The results in parentheses in Table II are from (8), in which  $x$  is the scale difference at the ninth magnitude derived from all the stars.

TABLE II  
SCALE-DIVERGENCE OF *Durchmusterung* FOR BRIGHT STARS

Mean <i>HPg</i>	<i>MW-HA 101</i>	No. Stars	<i>O-C</i>
8.00.....	+0.10	14	-0.02
8.74.....	.15	13	-.01
(9.00).....	(.20)	(80)	(+.02)
9.23.....	.12	26	-.07
9.78.....	+0.24	20	+0.01

The representation of the differences in the second column of Table II by the term  $+0.06 (HPg-6)$  is shown in the last column of the table. The residuals are satisfactorily small and show that the magnitudes of the bright stars of the *Durchmusterung*, after correction for color equation, are really in agreement with the scale of *Harvard Annals*, 71, No. 3. This follows from the fact that the divergence of the latter from the Mount Wilson photographic scale for  $m < 10$  was shown in *Contribution No. 98*<sup>1</sup> to have the form  $+0.061 (HPg-6)$ . Since the Mount Wilson results are on the international system, the complete reduction to this system of the bright stars in the areas of the *Durchmusterung* photographed with the Metcalf telescope before regrinding is therefore

$$+0.06 (DM-6) + 0.21 C, \quad (9)$$

which applies for  $m \leq 10$ .

Strictly speaking, a knowledge of this correction is not in itself of much importance because of the small number of stars to which it is applicable. The value of the discussion is in the information it gives as to the scale of the *Durchmusterung* and the color equation

<sup>1</sup> *Astrophysical Journal*, 41, 259, 1915.



of the instrument. The latter, it should be repeated, refers to data obtained before the objective was reground.

An attempt has been made to determine the effect of regrinding upon the color equation by discussing the sixteen areas photographed after this had been done. The color coefficient is perhaps a little smaller than was previously the case, but the result is very uncertain. The main conclusion is that the scale difference for these particular areas is about 0.2 mag. greater than that found above, an irregularity not easily explained.

### 3. CORRECTION FOR THE FAINT STARS OF THE *Durchmusterung*

As already mentioned, the faint stars are numerous enough to permit a direct comparison of scales for each area. Since the colors of individual stars are unknown, color equation cannot be allowed for, and what is obtained is the mean difference between the *Durchmusterung* scale, affected by color equation, and the international photographic scale. The corrections thus obtained will answer for a statistical treatment of the data; but used for individual stars, they may give results considerably in error.

The results for each of Areas 1-139 are in Table III, which gives the corrections, in hundredths of a magnitude, for the statistical reduction of the *Durchmusterung* values to the international system. An asterisk following the number of an area in the first column indicates regions photographed with the Metcalf telescope after regrinding; a dagger, those photographed with the Bruce telescope. All other areas were observed with the original objective of the Metcalf telescope. The arguments at the heads of the columns are *Durchmusterung* magnitudes. The numbers of stars given in the last column are the totals used in deriving the corrections. In rich regions these do not always represent the totals common to the two catalogues, although available data have not been omitted unless really superfluous. For example, all the stars brighter than the twelfth or thirteenth magnitude, which at best are none too numerous, were used. The tabular corrections themselves are the ordinates of smooth curves drawn through plots of the differences  $MW - HA$  101, which were made for each of the areas.



TABLE III

Area	Lat.	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	No. Stars
1.	+28°				+20	+34	+37	+41	+44	+48	+49	+50	+50	+50			40
2.	13			+47	47	48	38	32	26	18	11	4	2	-11			15
3.	17			49	49	48	40	48	40	37	33	27	20				74
4.	32			65	68	66	68	68	70	78	87	82	14				34
5.	46	+61		65	65	66	66	66	66	78	87	82	14				34
6.	32	+33		58	58	58	58	58	58	60	60	60	60				34
7.	38			58	58	58	58	58	58	60	60	60	60				34
8.	48			49	49	49	49	49	49	51	51	51	51				117
9.	-3			58	58	58	58	58	58	60	60	60	60				157
10.	+2			58	58	58	58	58	58	60	60	60	60				103
11.	13			51	51	51	51	51	51	53	53	53	53				23
12.	26			56	56	56	56	56	56	58	58	58	58				23
13.	41			56	56	56	56	56	56	58	58	58	58				23
14.	53			33	32	32	32	32	32	34	34	34	34				23
15.	57			30	30	30	30	30	30	32	32	32	32				23
16.	48			74	74	72	70	63	47	37	24	14	4				40
17.	33			74	71	67	58	49	45	44	42	37	30				35
18.	19			73	39	41	41	40	37	33	26	18	11				213
19.	+6			39	39	41	41	40	37	33	26	18	11				80
20.	-1			20	22	25	27	26	27	25	24	19	13				141
21.	18			41	41	42	41	40	40	40	40	41	41				85
22.	17			41	41	44	48	52	62	64	56	40	43				108
23.	13			19	38	51	53	47	42	39	40	40	43				100
24.	-7			48	50	53	52	50	44	37	38	43	41				88
25.	0			31	36	40	44	41	33	24	13	3	-12				110
26.	+8			40	48	46	40	33	25	22	21	25	28				81
27.	28			50	47	41	36	32	27	28	28	27	27				68
28.	39			31	40	46	48	50	48	44	41	35	29				52
29.	30			12	12	24	30	29	28	28	28	27	27				38
30.	60			28	26	24	20	18	17	16	13	10	6				34
31.	68			45	43	42	42	42	40	39	38	36	36				31
32.	72			45	43	42	42	42	40	39	38	36	36				26
33.	68			45	43	42	42	42	40	39	38	36	36				32
34.	59			45	43	42	42	42	40	39	38	36	36				30
35.	49			45	43	42	42	42	40	39	38	36	36				30
36.	39			45	43	42	42	42	40	39	38	36	36				43
37.	28			45	43	42	42	42	40	39	38	36	36				49
38.	18			45	43	42	42	42	40	39	38	36	36				57
39.	9			45	43	42	42	42	40	39	38	36	36				106
40.	0			45	43	42	42	42	40	39	38	36	36				206
41.	-8			45	43	42	42	42	40	39	38	36	36				118
42.	13			45	43	42	42	42	40	39	38	36	36				144
43.	16			45	43	42	42	42	40	39	38	36	36				112
44.	-32			45	43	42	42	42	40	39	38	36	36				72
				45	43	42	42	42	40	39	38	36	36				51

\*Observed with Metcalf Telescope after regrounding objective.





Examination of the curves shows that the accidental errors are large, and that the correction curves differ much from area to area. The average deviation of the plotted points from the curves, which represents the combined accidental error of the two catalogues, was formed for eleven areas, Nos. 1, 11, 21 . . . , 101, with results as follows:

$$\pm 0.27, 0.20, 0.18, 0.26, 0.22, 0.19, 0.22, 0.16, 0.19, 0.16, 0.16,$$

or an average of about  $\pm 0.20$  mag.

Since the probable error of a Mount Wilson magnitude, including scale error, ranges from 0.04 to 0.08 mag., according to the number of images measured, it follows that the larger part of the accidental error is in the *Durchmusterung* magnitude. This is to be expected, however, for the values of these magnitudes depend on one, or at most two, images measured on a single plate and reduced with standards whose values were found by a method which cannot be expected to give high precision.

The fluctuations in the correction curves from area to area represent the combined effect of scale errors in the catalogues. The Mount Wilson results are means based on six separate determinations of the scale in each area. The *Durchmusterung* depends on standard stars whose magnitudes were determined at Harvard with the aid of a relation between magnitude and scale reading of images found from three plates, taken from each box of twelve, and exposed on the Polar Sequence. The mean reduction curve for the three plates was used to determine the magnitudes of the standards on the nine remaining plates, which were exposed directly to the areas. Although the twelve plates were developed together, gradation differences from plate to plate seem to have affected the results appreciably.

In *Contribution* No. 287 it is shown that the mean color-index of the stars of the sixth photographic magnitude increases about 0.25 mag. in passing from the galaxy to the galactic pole, and that for the stars of the fourteenth magnitude or fainter the increase is probably larger. Since the color equation of the Metcalf telescope is  $+0.21 C$ , the total correction to the *Durchmusterung* should be

systematically larger for stars in high galactic latitudes than for those near the Milky Way.

To see whether this difference, apparently of the order of 0.10 mag., can be detected, the corrections in Table III for the eighty-three areas observed with the Metcalf telescope in its original state were collected in four groups arranged according to the galactic latitude given in the second column of the table. The mean corrections for all the areas are in the second column of Table IV, while the deviations for each of the four groups, in the sense, Group *minus* Mean, are in the columns immediately following. The figures in parentheses indicate the number of areas included in each value.

The deviations for the second and third groups are small and show no important systematic effect. The first and third, however, diverge systematically, but in the direction opposite to that to be expected from the known difference in the average color of stars in high and low latitudes. Since the color equation is positive and the stars near the galactic pole are redder than those in the Milky Way, the corrections for the fourth group should be the largest, whereas this is actually the case for the first group. There is no obvious explanation of this anomaly.

Mean corrections for the areas observed with the Metcalf telescope after regrinding and with the Bruce telescope were also formed. Their deviations from the corrections in the second column of Table IV are in the seventh and eighth columns, respectively.

With the aid of (1), which gives the mean color-index for a grouping according to photographic magnitude, and the color equation  $+0.211 C$  from (8), it is possible to separate the scale-divergence from the values of the total mean correction in the second column of Table IV. The color corrections thus found, which are to be subtracted, are in the ninth column, and the resulting scale-divergence from the international system, in the tenth column. The following columns, on the other hand, give the scale reduction to the international system for the Harvard magnitudes of the Polar Sequence stars in *Harvard Annals*, 71, No. 3, after correction for color. These quantities are smoothed values of the differences  $MW - HH_w$ , derived from Table III of *Contribution* No. 98.<sup>1</sup> The differences in the last col-

<sup>1</sup> *Astrophysical Journal*, 41, 259, 1915.

TABLE IV  
COMPARISON OF SYSTEMATIC CORRECTIONS

Harv.-Gron. DM Mag.	Mean Total Correction	Correction for Groups				Metcalf Reground	Bruce Telescope	Color Corrn.	Mean Scale Corrn.	$P_z -$ HA 71	Red. to HA 71
		0°-15°	16°-28°	32°-49°	53°-85°						
10.0.....	+0.40(6)	0(2)	- 8(2)	+ 8(2)	.....	.....	0(2)	0.12	+0.28	+0.24	+0.04
10.5.....	.38(30)	- 12(0)	+ 14(8)	- 2(0)	0(4)	.....	6(4)	.12	.26	.24	.02
11.0.....	.37(71)	- 2(10)	7(16)	+ 1(22)	- 7(14)	+ 7(4)	+ 7(10)	.13	.24	.24	.00
11.5.....	.41(86)	2(21)	4(10)	1(22)	9(18)	- 2(12)	2(24)	.14	.27	.24	.03
12.0.....	.41(82)	6(21)	4(20)	0(22)	11(10)	- 1(14)	1(36)	.15	.26	.24	.02
12.5.....	.41(82)	8(21)	2(20)	1(22)	11(10)	0(15)	0(40)	.16	.25	.24	.01
13.0.....	.39(83)	9(21)	0(21)	1(22)	10(10)	+ 2(15)	1(40)	.16	.23	.24	-
13.5.....	.35(83)	9(21)	0(21)	0(22)	8(10)	5(15)	1(40)	.17	.18	.24	.06
14.0.....	.32(83)	8(21)	- 1(21)	0(22)	6(10)	4(16)	- 1(40)	.18	.14	.24	.10
14.5.....	.30(83)	7(21)	1(21)	1(22)	6(10)	4(16)	5(40)	.18	.12	.24	.12
15.0.....	.27(83)	7(21)	- 1(21)	1(22)	6(10)	8(16)	8(40)	.19	.08	.23	.15
15.5.....	.21(78)	9(21)	0(20)	3(20)	6(17)	13(16)	- 3(38)	.20	.01	.20	.10
16.0.....	+ .16(55)	12(17)	+ 1(11)	- 4(12)	10(15)	12(16)	+ 2(37)	.20	.....	.16	.20
16.5.....	- 0.08(7)	+ 30(1)	+ 6(2)	+ 12(1)	- 17(3)	+ 27(11)	+ 17(22)	0.21	- 0.29	+ 0.13	- 0.42

umn represent the mean correction for scale required to reduce the *Durchmusterung* to *Harvard Annals*, 71, No. 3. To about magnitude 13.5 the corrections are negligible, and show that the scales are identical, as was intended. For the fainter stars, however, there is an appreciable divergence. This seems to be less pronounced for the areas observed with the Metcalf reground and the Bruce telescope, although the uncertain color equations for these instruments make quantitative estimates unreliable.

From this discussion it is clear that the corrections for individual areas given in Table III fluctuate to such an extent as to obscure the differential effect of color and make it unwise to attempt to use a mean reduction curve except in certain special cases. For general statistical purposes it is probably safer to correct the scales of the separate areas by means of the data in Table III. Finally, it should be remarked that only a small part of these fluctuations can originate in the zero-point errors of the Mount Wilson magnitudes. Great care was taken to make these results homogeneous in the matter of zero-point,<sup>1</sup> and the internal agreement is such as to indicate that the errors affecting the individual areas are very small.

MOUNT WILSON OBSERVATORY  
October 1924

<sup>1</sup> See *Third Report on the Progress and Plan of the Selected Areas*, Second Appendix, Groningen, 1923.



# THE ORBITS OF FOUR SPECTROSCOPIC BINARIES<sup>1</sup>

By R. F. SANFORD

## ABSTRACT

*Elements of spectroscopic binaries.*—Orbital elements have been derived for the four binaries, Boss 1131, H.D. 101206, B.D. +55°2215, and  $\beta$ GC 9818 S (Tables IV, VI, VIII, and X).

The spectrum of B.D. +55°2215 bears a resemblance to that of some of the Cepheid variables, but has elements which differ widely from those characterizing these variables. Its light is not known to vary.

Measures of the northern component of the visual binary  $\beta$ GC 9818 give a radial velocity in substantial agreement with that here derived for the southern component.

*Dwarf spectroscopic binaries.*—The dwarf binaries H.D. 101206 and  $\beta$ GC 9818 S have periods and eccentricities which conform with those found for the majority of dwarf spectroscopic binaries, while Boss 1131, another dwarf, has neither a short period nor a small eccentricity. The velocities of the center of mass for these dwarfs are not excessively large, although the values for Boss 1131 and  $\beta$ GC 9818 S are somewhat above the average.

This paper deals with the orbits of the four spectroscopic binaries whose designations, co-ordinates for 1900, apparent and absolute magnitudes, proper motions, parallaxes, and number of orbital revolutions included by their respective observations are listed in Table I. For the first three the absolute magnitudes are those determined spectroscopically by Adams and his associates, from which the

TABLE I

Name	Vis. App. Mag.	$\alpha$ (1900)	$\delta$ (1900)	Spectral Class	Vis. Abs. Mag.	$\mu$	$\pi_{sp}$	No. Rev.
Boss 1131.....	6.8	4 <sup>h</sup> 42 <sup>m</sup> 8	+18°33'	G0	5.0	0".440	0".044	88
H.D. 101206...	8.2	11 33.7	42 52	K3	5.7	0.49	.050 $\pm$	114
B.D. +55°2215..	6.5	19 29.2	55 31	cG6	-0.9 $\pm$	.....	.003 $\pm$	12
$\beta$ GC 9818 S....	7.5	19 56.6	+12 28	F3	5.5 $\pm$	.....	0.040	104

parallaxes follow by the well-known formula. In the last case the parallax given is one derived by Russell from considerations of visual binaries. The absolute magnitude has been computed from this parallax. All four stars have been previously announced as

<sup>1</sup> Contributions from the Mount Wilson Observatory No. 291.



spectroscopic binaries.<sup>1</sup> Reference may be made to the introduction to *Mount Wilson Contributions*, No. 201,<sup>2</sup> for general remarks which are also applicable to points in this paper. The details for each binary follow.

#### BOSS 1131<sup>3</sup>

BOSS 1131 has a spectrum classed as G5 at Harvard<sup>4</sup> and Go by the Mount Wilson observers, which gives an absolute magnitude of +5.0, thus placing it among dwarfs. Only the lines of the primary star are present.

Eight plates secured in the interval 1913-1918 definitely established variation in its radial velocity. These plates, however, were so widely scattered as to give no clue to the period. Observations begun in 1920 showed changes in velocity that, for a time, were ascribed to a period of about a day. Following this assumption the writer has accumulated a considerably larger number of plates for this spectroscopic binary than for the others he has investigated at this observatory. This accounts for the bunching of observations at certain times.

Table II lists the data for forty-three spectrograms obtained with one-prism dispersion and an 18-inch camera in conjunction with both the 60-inch and 100-inch reflectors. Numerous trials finally led to the much longer period of 45.454 days, which affords a satisfactory assemblage of all observations at a single epoch. The possibility of a superposed short-period velocity-variation will be alluded to later. The above-stated period cannot be changed more than 0.01 day without, in certain cases, seriously disturbing the distribution of observed velocities upon the velocity-curve. The number of orbital revolutions covered by the observations, together with the possibility of a superposed short-period variation, seemed sufficient justification for not including this element for correction by least squares.  $P = 45.454$  days is therefore considered determined.

<sup>1</sup> BOSS 1131, *Publications of the Astronomical Society of the Pacific*, **31**, 41, 1919. The other three stars, *ibid.*, **36**, 137, 1924.

<sup>2</sup> *Astrophysical Journal*, **53**, 201, 1921.

<sup>3</sup> B.D.+18°734; A. G. Berl. A 1309; Pi 190, H.D. 30455.

<sup>4</sup> The *Henry Draper Catalogue* states that H $\delta$  is unusually strong. Slit spectrograms do not bear this out.

To facilitate the least-squares solution and minimize any disturbance from a short-period variation, normal places were formed

TABLE II  
OBSERVATIONS OF BOSS 1131

Plate No.	Date	G.M.T.	Phase	Velocity	O-C	Wt.
				km/sec.	km/sec.	
$\gamma$ 2110.....	1913 Feb. 13	16 <sup>h</sup> 33 <sup>m</sup>	9 <sup>d</sup> 463	+44.9	- 3.1	.....
3116.....	1914 Feb. 6	16 39	3.833	64.9	+ 4.9	.....
3904.....	Nov. 25	20 52	23.288	41.2	- 6.3	.....
3972.....	Dec. 27	20 49	9.832	51.0	+ 3.5	.....
4061.....	1915 Feb. 25	17 33	24.241	47.8	- 0.2	.....
5231.....	1916 Nov. 8	22 28	10.089	35.0	-12.6	.....
5486.....	1917 Jan. 31	18 15	3.005	62.3	+ 0.6	.....
7621.....	1918 Dec. 12	21 10	1.316	69.5	+ 1.5	.....
9679.....	1920 Oct. 27	22 08	4.547	52.8	- 4.1	1.00
9683.....	Oct. 28	20 25	5.478	53.7	- 0.8	1.00
C 740.....	Oct. 31	23 37	8.610	53.0	+ 4.9	1.00
$\gamma$ 9714.....	Nov. 2	22 52	10.578	45.9	- 1.4	1.00
9728.....	Nov. 20	19 45	28.386	51.2	+ 0.8	1.00
C 806.....	Dec. 18	17 40	10.906	48.2	+ 1.2	1.00
$\gamma$ 9806.....	Dec. 22	18 21	14.935	46.3	+ 0.5	1.00
9850.....	Dec. 30	15 17	22.808	47.6	+ 0.3	1.00
9862.....	1921 Jan. 21	15 25	44.822	69.6	- 3.0	1.00
9872.....	Jan. 24	18 55	2.505	62.0	- 1.4	0.50
9928.....	Feb. 20	18 04	29.470	46.2	- 5.0	1.00
9933.....	Feb. 21	16 02	30.385	52.8	+ 0.8	0.25
C 895.....	Feb. 22	15 43	31.373	51.7	- 1.2	.50
900.....	Feb. 23	15 25	32.358	53.7	- 0.2	0.50
903.....	Feb. 24	14 55	33.339	59.2	+ 4.2	1.00
$\gamma$ 9996.....	Mar. 16	15 51	7.923	52.1	+ 2.3	1.00
10004.....	Mar. 17	15 40	8.916	42.8	- 6.0	1.00
C 946.....	Mar. 19	16 12	10.938	49.4	+ 2.4	0.50
954.....	Mar. 20	15 59	11.929	46.6	0.0	1.00
$\gamma$ 10675.....	Dec. 7	17 00	1.247	68.8	+ 0.8	1.00
10680.....	Dec. 7	23 22	1.512	63.3	- 3.8	1.00
10751.....	1922 Jan. 12	17 37	37.274	60.4	- 0.4	1.00
C 1525.....	Jan. 13	19 37	38.357	60.8	- 2.1	0.50
1532.....	Jan. 14	20 23	39.388	68.6	+ 3.6	1.00
$\gamma$ 10764.....	Jan. 15	15 29	40.184	65.0	- 1.6	1.00
10770.....	Jan. 15	21 17	40.384	68.0	+ 1.0	1.00
11458.....	Dec. 5	16 31	0.595	79.3	+ 9.3	1.00
11460.....	Dec. 5	20 42	0.770	61.8	- 7.7	0.50
11464.....	Dec. 6	0 1	0.907	78.2	+ 9.2	0.25
C 2037.....	1923 Jan. 1	21 35	27.806	50.8	+ 0.8	1.00
12006.....	Sept. 1	23 55	43.634	75.8	+ 3.0	1.00
C 2457.....	Sept. 27	0 35	23.207	46.6	- 0.9	1.00
$\gamma$ 12260.....	Oct. 30	22 21	11.660	44.5	- 2.2	1.00
12300.....	Nov. 25	17 54	37.478	58.9	- 2.3	1.00
12403.....	1924 Jan. 17	17 58	45.024	+68.0	- 4.3	1.00

by combining, in a single mean, series of observations taken close together, due consideration being given to the nature of the veloc-

ity variation at the particular epoch. The observations made before 1920 were not included. The seventeen normals thus formed (Table III) gave, by Russell's method, preliminary values for the

TABLE III  
NORMAL PLACES—BOSS 1131

No.	Phase	Limits of Phase	Velocity	O—C	Wt.
			km/sec.	km/sec.	
1.....	0 <sup>d</sup> 692	0 <sup>d</sup> 60—0 <sup>d</sup> 91	+74.1	+4.3	1.75
2.....	1.379	1.25—1.51	66.0	-1.6	2.00
3.....	2.505	.....	62.0	-1.4	0.50
4.....	7.303	4.55—10.58	51.3	+0.6	4.00
5.....	9.782	7.92—11.92	47.5	-0.4	3.50
6.....	11.660	.....	44.5	-2.2	1.00
7.....	12.921	10.91—14.94	47.2	+1.0	2.00
8.....	22.808	.....	47.6	+0.3	1.00
9.....	23.207	.....	46.6	-0.9	1.00
10.....	27.806	.....	50.8	+0.8	1.00
11.....	28.386	.....	51.2	+0.8	1.00
12.....	31.468	29.47—33.34	52.7	-0.3	3.25
13.....	37.478	.....	58.9	-2.3	1.00
14.....	39.202	37.27—40.38	65.0	+0.4	4.50
15.....	43.634	.....	75.8	+3.0	1.00
16.....	44.822	.....	69.6	-3.0	1.00
17.....	45.024	.....	+68.0	-4.3	1.00

TABLE IV  
ELEMENTS OF BOSS 1131

	PRELIMINARY ELEMENTS	CORRECTIONS		FINAL ELEMENTS
		First. Sol.	Second Sol.	
<i>P</i> .....	45 <sup>d</sup> 454	.....	.....	45 <sup>d</sup> 454
<i>e</i> .....	0.55	-0.160	+0.001	0.391 ± 0.038
<i>ω</i> .....	39 <sup>o</sup>	-13.96	+1.44	26 <sup>o</sup> 48 ± 6 <sup>o</sup> 52
<i>K</i> .....	14.0	-1.16	+0.73	13.57 ± 0.45 km/sec.
<i>T</i> .....	J.D. 2423030.89	-0.445	+0.016	3030.461 ± 0.482
<i>γ</i> .....	+55.0	-0.18	-0.16	+54.66 km/sec.
<i>a</i> sin <i>i</i> .....	.....	.....	.....	7,810,500 km
<i>m</i> <sub>1</sub> <sup>3</sup> sin <sup>3</sup> <i>i</i>	.....	.....	.....	0.0092 ⊙
<i>(m</i> + <i>m</i> <sub>1</sub> ) <sup>2</sup>	.....	.....	.....	.....

five remaining elements. Two least-squares solutions resulted in two sets of corrections to be applied to the provisional elements, and gave data for deriving the probable errors. Table IV gives the preliminary elements, the two sets of corrections, and the final ele-

ments with their probable errors, derived from a probable error  $\pm 1.9$  km/sec. for a normal place of unit weight;  $a \sin i$  and the mass factor have been added.

Figure 1 shows the computed radial velocity-curve with the seventeen normal places plotted as circles with radius 1.2 km/sec. Individual observations are too numerous to admit plotting in this manner.

The column headed "O-C" in Table III gives the differences between the value of the velocity of a normal place and that

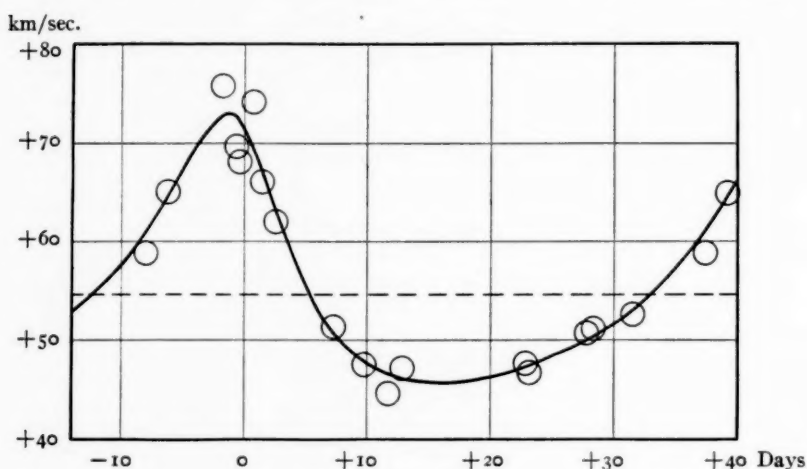


FIG. 1.—Velocity-curve of Boss 1131

computed by the final elements. The corresponding quantity in Table II has been found by scaling off the difference between the individual velocities and the computed velocity-curve.

The largest of the latter residuals,  $-12.6$  km/sec., is from plate  $\gamma$  5231, whose quality is decidedly poor. The others are all numerically less than 10 km/sec., and, by themselves, perhaps would not be seriously questioned, even for a spectrum as satisfactory as that of this star. The disturbing feature is best displayed by such runs of observations as those in the intervals October 27 to November 2, 1920, and December 5-6, 1922, where the range in velocity seems too great to be explained by the dispersion employed. The second set, however, involves two plates of low weight. An attempt to fit the

residuals to a short period has not been successful, as is perhaps to be expected when the variations themselves are at most but slightly greater than those legitimately to be anticipated from the dispersion employed. Hence the existence of this type of variation remains unsettled.

H.D. 101206<sup>1</sup>

This star was observed for absolute magnitude because its known proper-motion indicated low luminosity. The discordant values for radial velocity from the first three plates led to the discovery of its binary character. The spectrum, at all times single, is of spectral class K<sub>3</sub>.

TABLE V  
OBSERVATIONS OF H.D. 101206

Plate No.	Date	G.M.T.	Phase	Velocity	O-C	Quality
				km/sec.	km/sec.	
C 420.....	1920 May 10	18 <sup>h</sup> 01 <sup>m</sup>	4 <sup>1</sup> 332	+19.1	-0.2	Poor
863.....	1921 Jan. 24	22 40	5.191	+26.2	-2.9	Good
985.....	Apr. 11	19 27	4.556	+10.9	-11.3	Good-
1125.....	July 13	16 02	6.997	+39.5	-1.8	Good*
1697.....	1922 May 12	16 46	0.026	-20.4	-7.5	Poor
1740.....	June 10	17 40	3.231	+5.2	+0.3	Fair-
γ 11499.....	1923 Jan. 3	0 22	2.842	+11.1	+10.9	Poor
11503.....	Jan. 4	0 00	3.827	+21.2	+8.5	Poor focus
C 2101.....	Feb. 1	0 34	6.018	+34.2	-2.1	Fair+
γ 11749.....	May 2	18 42	6.356	+40.6	+2.4	Fair+
C 2238.....	May 4	18 30	8.348	+42.8	+2.6	Fair-
γ 11761.....	May 5	17 45	9.317	+27.6	-6.0	Fair+
11767.....	May 6	17 03	10.288	+18.4	-3.8	Fair-
C 2281.....	May 31	17 09	9.458	+31.2	-1.0	Good
2327.....	June 27	16 30	10.598	+23.0	+5.2	Fair-
γ 11898.....	June 28	16 40	11.604	+4.4	+1.7	Good*
12408.....	1924 Jan. 18	0 36	8.268	+35.6	-5.0	Good
12413.....	Jan. 19	0 38	9.269	+35.0	+1.0	Fair+
C 2640.....	Jan. 19	21 04	10.122	+22.8	-1.6	Fair+
2644.....	Jan. 20	21 28	11.138	+10.6	+0.9	Good
2687.....	Feb. 18	0 06	0.497	-9.0	+6.2	Fair
γ 12507.....	Feb. 20	21 39	3.395	+0.8	-6.3	Good*
C 2751.....	Apr. 18	16 36	9.519	+32.4	+0.8	Fair
2787.....	May 19	17 00	1.783	-13.8	-2.4	Good-

\* Seven-inch camera.

The apparent magnitude, spectral class, and winter observing season form a combination unfavorable to obtaining uniformly good

<sup>1</sup> B.D.+43°2135; A.G. Bonn 8141; Furihjelms 36 (*Finska Vetenskap-Soc. Verhand.*, 59, 1917); Cin. 1435.

exposures. Some plates have been made at the 60-inch reflector with a widened slit, and three were obtained with the dispersion furnished by a 7-inch camera on the one-prism spectrograph. The quality is in several cases not of the best, as is indicated in the last column of Table V, which gives the observational data for the twenty-four plates obtained. The best grouping for the velocity-curve was obtained with a period of 12.9167 days, which can hardly be in error more than 0.002 day since 114 orbital revolutions separate the first and last observations. Provisional elements obtained in the usual way and then corrected by inspection appear in Table VI. Since they

TABLE VI

ELEMENTS FOR H.D. 101206

$P$ .....	12 <sup>d</sup> 9167	$T$ .....	J.D. 2423549.34
$e$ .....	0.08	$\gamma$ .....	+15.3 km/sec.
$\omega$ .....	154°0	$a \sin i$ .....	5,090,000 km
$K$ .....	28.8 km/sec.	$\frac{m_1^3 \sin^3 i}{(m+m')^2}$ .....	0.0316 ☉

reveal no points of more than ordinary interest it did not seem worth while to seek any refinement in the values of the elements by the considerably increased labor involved in obtaining more and better plates. The residuals in the sixth column of Table V are quite satisfactory for the quality and number of plates involved, and do not seem to justify the extra effort necessitated by a least-squares solution.

Figure 2 shows the velocity-curve derived from the elements in Table VI. The barred circles represent observed velocities derived from the three spectrograms of low dispersion.

B.D. +55°2215<sup>1</sup>

This star has been classified in the *Henry Draper Catalogue* as K2+A3, a composite spectrum, and assigned two numbers. The spectral class derived at Mount Wilson is about cG6, which not only accounts for the abnormal strength of the hydrogen lines, probably the reason for calling the spectrum composite, but is also consistent with the fact that the hydrogen lines seem to share the Doppler

<sup>1</sup> A.G. Hels. 10554; H.D. 184398-9.

shift of the other lines. The spectrum resembles that of X Cygni near minimum light, and closely duplicates that of Boss 1074, which has *c*-characteristics and a radial-velocity variation in a period which, although pretty certainly long, has not yet been determined. The spectral lines of B.D. +55°2215 are at all times single, and do not change in character to any notable extent.

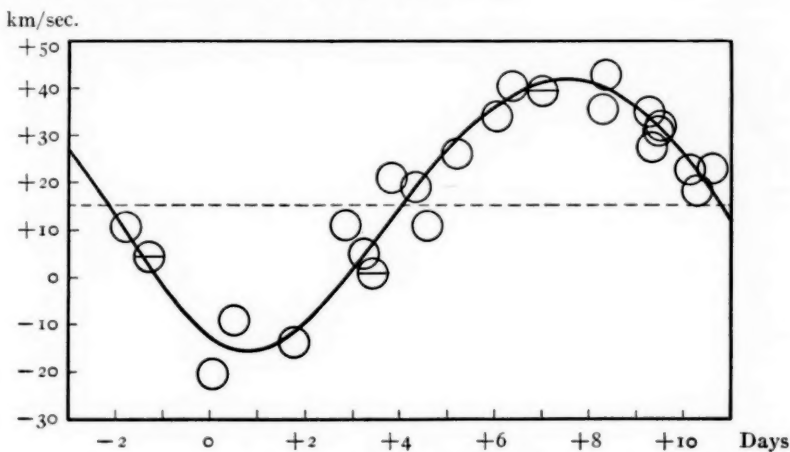


FIG. 2.—Velocity-curve of H.D. 101206

The measures of the second plate showed an undoubtedly variable radial velocity which was confirmed by the next two plates. Since May 1, 1920, the star has been under observation each season, and at present fifty spectrograms of suitable quality are available (Table VII). This number is somewhat larger than usual because of a suspected short-period variation, superposed upon the period of approximately 110 days. The shorter period was not confirmed—in fact was only suspected on the basis of some measures of weakly exposed plates. Considerable effort was expended in deriving the period (108.5707 days), which can hardly admit of a change as great as 0.05 day, and which has been adopted as final.

The points in Figure 3 represent the observed velocities collected with this period. Russell's method, with subsequent adjustment, partly by least squares and partly by trial, gave elements which define the velocity-curve shown in Figure 3. The representation is satisfactory, and these elements are adopted and given in Table VIII.



TABLE VII  
OBSERVATIONS OF B.D.+55°2215

Plate No.	Date	G.M.T.	Phase	Velocity	O-C
				km/sec.	km/sec.
$\gamma$ 9170.....	1920 May 1	23 <sup>h</sup> 32 <sup>m</sup>	24 <sup>d</sup> 587	-22.7	+4.6
10197.....	1921 June 13	21 45	106.800	-0.2	+1.5
10300.....	July 20	22 56	35.279	-26.6	-2.3
10351.....	Aug. 13	19 36	59.140	-1.4	-0.6
10534.....	Oct. 12	16 18	10.431	-17.0	+0.2
10590.....	Nov. 6	16 17	35.430	-25.6	-1.4
C 1570.....	1922 Feb. 16	0 34	28.205	-27.8	-0.7
1577.....	Feb. 17	1 06	29.227	-23.2	+3.8
1586.....	Feb. 18	1 15	30.233	-21.4	+5.2
$\gamma$ 10863.....	Mar. 7	0 42	47.210	-17.0	-2.8
C 1644.....	Apr. 6	23 42	78.169	+14.2	-1.6
$\gamma$ 11040.....	May 11	22 29	4.544	-17.4	-6.5
C 1690.....	May 12	22 23	5.542	-7.8	+4.2
$\gamma$ 11095.....	June 7	20 36	31.469	-23.6	+2.4
11099.....	June 8	18 08	32.367	-22.2	+3.7
C 1736.....	June 9	18 24	33.378	-32.2	-6.8
1741.....	June 10	19 42	34.432	-25.0	-0.1
$\gamma$ 11120.....	July 2	18 43	56.391	-4.2	-0.2
11124.....	July 2	23 38	56.596	-7.8	-4.5
11126.....	July 3	18 17	57.373	+3.3	+5.9
11130.....	July 3	23 39	57.596	-1.8	+0.7
11134.....	July 4	22 50	58.562	+0.6	+2.2
11200.....	July 16	20 42	70.473	+11.1	+0.5
11220.....	Aug. 5	16 04	90.280	+15.9	+0.4
11223.....	Aug. 5	20 40	90.472	+15.2	-0.2
11270.....	Aug. 30	16 51	6.742	-10.8	+2.6
C 1850.....	Sept. 1	15 12	8.673	-10.2	+5.8
$\gamma$ 11300.....	Sept. 3	16 40	10.734	-20.2	-2.0
C 1864.....	Sept. 5	17 20	12.762	-20.2	-1.2
1872.....	Sept. 8	20 27	15.892	-24.4	-2.0
1919.....	Oct. 4	14 55	41.662	-19.2	+0.7
$\gamma$ 11355.....	Oct. 5	15 49	42.699	-18.6	-0.1
11369.....	Oct. 7	17 01	44.749	-21.8	-4.8
11457.....	Dec. 5	15 10	103.672	+9.2	+6.7
C 2235.....	1923 May 3	22 00	35.822	-21.6	+2.5
$\gamma$ 11763.....	May 5	20 50	37.767	-22.9	+0.1
11770.....	May 6	20 45	38.763	-22.3	-0.1
11874.....	June 23	23 19	86.871	+15.8	-1.0
C 2323.....	June 26	18 22	89.664	+16.8	+1.0
$\gamma$ 11907.....	June 29	20 10	92.739	+20.5	+6.3
11930.....	July 4	21 53	97.811	+9.4	-0.5
12173.....	Oct. 3	15 31	79.974	+20.0	+3.7
12244.....	Oct. 28	16 58	105.035	+2.6	+1.6
12299.....	Nov. 25	16 00	24.424	-31.1	-3.8
C 2786.....	1924 May 18	23 42	91.176	+13.2	-1.8
2790.....	May 19	21 59	92.102	+14.0	-0.6
2835.....	June 17	23 31	12.594	-25.9	-6.2
$\gamma$ 12784.....	July 13	17 06	38.329	-23.1	-0.4
12790.....	July 14	16 37	39.309	-24.2	-2.4
12796.....	July 15	17 35	40.348	-22.0	-1.0



The residuals O-C in Table VII have been scaled from the velocity-curve of Figure 3. The agreement is typical of plates of this dispersion and spectral character, and gives no indication of a superposed velocity variation of short period.

km/sec.

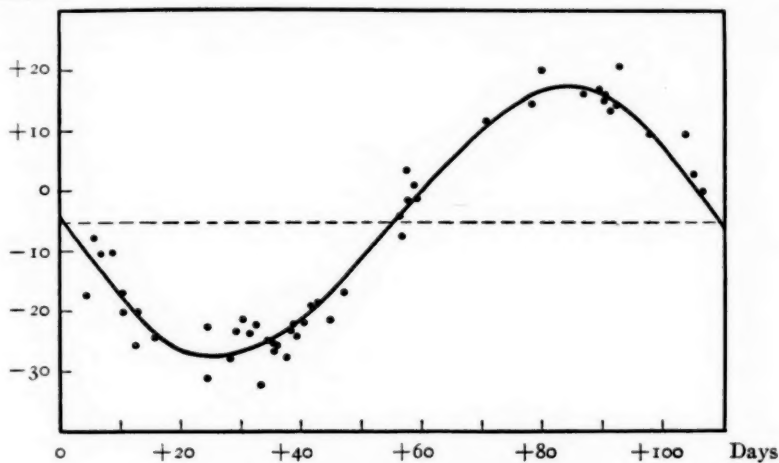


FIG. 3.—Velocity-curve of B.D.+55°2215

Aside from the spectrum, there is little to lend support to the Cepheid character of this star; for it is not known to vary in magnitude, and its orbital elements in several respects are markedly

TABLE VIII

ELEMENTS B.D.+55°2215

$P$ .....	108 <sup>d</sup> 5707	$T$ .....	J.D. 2423375.760
$e$ .....	0.054	$\gamma$ .....	-5.2 km/sec.
$\omega$ .....	87°0	$a \sin i$ .....	33,040,000 km.
$K$ .....	22.1 km/sec.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$ .....	0.1222 $\odot$

different from the so-called elements of the average Cepheid variable. In addition, the period-luminosity law would demand a luminosity far greater than that consistent with the usual spectral considerations.

$\beta$ GC 9818 S<sup>1</sup>

This star is the southern component of a visual pair whose position-angle and distance according to Burnham's *General Catalogue* remained sensibly constant at  $350^\circ$  and  $4''.4$ , respectively, during the interval 1866-1905. The spectral class is F<sub>3</sub> and is at all times that of the primary. The marked change in radial velocity revealed by the second plate was borne out by the later plates. Twenty plates

TABLE IX  
OBSERVATIONS OF  $\beta$ GC 9818 S

Plate No.	Date	G.M.T.	Phase	Velocity	O - C	Quality
				km/sec.	km/sec.	
$\gamma$ 11883.....	1923 June 24	23 <sup>h</sup> 21 <sup>m</sup>	3 <sup>d</sup> 862	-64.2	-0.2	f+*
11957.....	July 24	23 28	2.578	-64.7	+7.0	f-
12114.....	Sept. 23	17 55	0.770	-2.8	-0.5	f+
12121.....	Sept. 24	16 53	1.727	-36.5	-2.2	g
12138.....	Sept. 27	16 27	0.238	-17.4	-6.0	f+
12243.....	Oct. 28	15 28	4.387	-32.5	-2.0	g
12248.....	Oct. 29	15 14	0.898	-6.0	-4.0	g
12256.....	Oct. 30	15 30	1.910	-49.8	-6.3	g
12571.....	1924 Apr. 16	23 42	1.399	-12.3	+6.4	g
12576.....	Apr. 17	23 10	2.377	-73.0	-8.6	f-
C 2754.....	Apr. 18	23 42	3.399	-73.5	+7.3	f+
12648.....	May 16	22 08	0.045	-20.4	+0.1	f
12652.....	May 17	21 45	1.024	-5.2	0.0	f+
C 2701.....	May 19	22 56	3.078	-91.8	-9.5	vg
2834.....	June 17	22 08	0.673	-6.6	-6.5	g
12785.....	July 13	18 36	4.261	-52.0	-12.0	vg
12791.....	July 14	17 52	0.760	-0.9	-0.8	f
12797.....	July 15	18 54	1.804	-33.3	+5.2	g
12939.....	Sept. 10	17 40	0.646	+7.6	+7.6	g
12943.....	Sept. 11	17 24	1.634	-29.8	+0.1	g

\* f = fair, g = good, vg = very good.

obtained between June, 1923, and September, 1924, seemed to be of such quality and distribution as to define the elements of the orbit with a precision adequate for this case, which presents no particular interest. The period, 4.4698 days, found by trial, does not admit of a change greater than 0.002 day, and has been adopted. The elements derived, as in the preceding cases, are based upon the observations in Table IX. After slight modification by inspection, the comparison of the resulting curve with the observations (Fig. 4) and a consideration of the magnitude of the residuals in Table IX made it seem

<sup>1</sup>  $\Sigma$  2613; B.D. +10° 4143; A.G. Leipz. 7630; H.D. 189783.

unlikely that a least-squares solution would afford any material gain. The adopted elements appear in Table X.

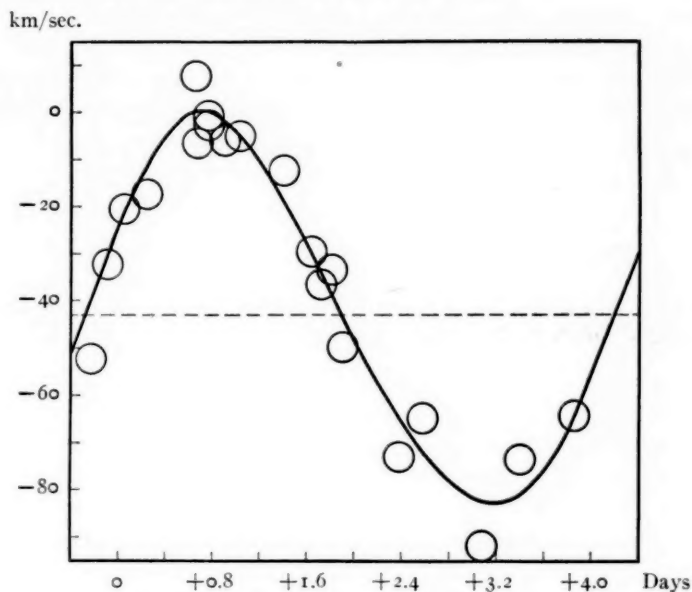


FIG. 4.—Velocity-curve of  $\beta$ GC 9818 S

The radius of the circles representing the observations in both Figures 2 and 4 is 3 km/sec., a quantity slightly larger than the probable error to be expected in such cases.

TABLE X

ELEMENTS  $\beta$ GC 9818 S

$P$ .....	4 <sup>d</sup> 4696	$T$ .....	J.D. 2423802.192
$e$ .....	0.10	$\gamma$ .....	-43.0 km/sec.
$\omega$ .....	294 $^{\circ}$ .5	$a \sin i$ .....	2,526,000 km
$K$ .....	41.3 km/sec.	$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$ .....	.00322 $\odot$

Two plates of the northern component show its spectrum to be nearly the same as that of the southern component. Their measures agree as well as could be expected and give -39.3 km/sec. for the radial velocity, a difference of -3.7 km/sec. from  $\gamma$  determined above. The probable errors will account for this difference.

In conclusion, it may be noted that three of these stars may be classified as dwarfs. The second and fourth are additional examples of dwarf binaries whose periods and eccentricities are both small as compared with those of giants—a fact to which attention has been called.<sup>1</sup> The first star, however, is a dwarf having both a period of considerable length and a decided eccentricity. It is also perhaps worth noting that the values of  $\gamma$  for both Boss 1131 and  $\beta$ GC 9818 S are rather large.

MOUNT WILSON OBSERVATORY  
November 1924

<sup>1</sup> *Mt. Wilson Contr.*, No. 251, 445; *Astrophysical Journal*, 56, 454, 1922; *Lick Observatory Bulletins*, 11, 132, 1924.

## REVIEWS

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*Helmholtz' Treatise on Physiological Optics.* Translated from the third German edition, edited by JAMES P. C. SOUTHALL, Professor of Physics in Columbia University. Published by the Optical Society of America.<sup>1</sup> Volume I. Pp. xxi+482. Figs. 149, portrait. \$7.00.

It has been considered witty of late years to say that the only thing to do with scientific books out of print for a dozen years or more is to feed them to the furnace and thereby lessen our coal bills. It may seem strange that a book written much over half a century ago should reach a third edition, and that a translation of it should appear in America, the land credited with love of the dollar rather than with devotion to research. Above all, it is noteworthy that the young American Optical Society, with its comparatively small membership, should venture to reproduce a classical work, in another language and another garb.

It is not too much to imagine that when the story of the development of scientific research in America is told, after another half-century, no better evidence will be cited by the historian than the enthusiasm in disseminating great truths which prompted this translation of Helmholtz' *Physiological Optics*.

It would be futile to attempt to review, within ordinary limits, a monumental work like this; even to compare this translation with the third edition, published in 1909, would be a difficult and delicate task. But one can express for himself and his colleagues the grateful appreciation of all who may wish to consult this encyclopedic treatise on optical questions. The editor of the translation showed unusual judgment in obtaining the co-operation of a dozen experts, each an acknowledged authority in the aspect of the work assigned to him. Dr. Southall has also been particularly fortunate in having the friendly co-operation of two out of the three editors of the third edition—Professors Gullstrand and von Kries. He gives due credit to those who have assisted him, mentioning particularly his wife and Miss Ruth Townsend, who "copied the entire

<sup>1</sup>It is requested by the Society that orders for the three volumes be placed with Professor F. K. Richtmyer, Secretary, Cornell University, Ithaca, New York. It is expected that the third volume will be ready by June, 1925.

manuscript thus far, besides taking time to perform some of Helmholtz' experiments as she went along."

It is impossible to consider in detail any one of the many points of interest. The laws of optical imagery would attract mathematicians; monochromatic and other aberrations would appeal to the optical manufacturer or the photographer; entoptical phenomena to the astronomer, and the illumination of the eye to the ophthalmologist, especially as dealt with in the exhaustive appendices by Gullstrand. But it is probable that no portions of this first volume will be referred to more frequently or read with more interest than those which relate to the mechanism of accommodation. That problem presents itself in some way, like Banquo's ghost, no matter what phase of the general subject is under consideration; for in spite of the battles between Tscherning and von Hess, which have been fought in the pages of the *Klinische Monatsblätter* and elsewhere, opinions still differ. Laurels may still be won in this field of investigation.

In a certain way this new edition in English is not an absolute necessity, for practically every competent student of astronomy, of ophthalmology, or related branches of physics has a sufficient knowledge of German to dig out the original. But the digging was often hard. For even Helmholtz' usually clear style is marred occasionally by qualifying clauses which, like meddling relatives, produce such an apparent estrangement between the substantive and the verb as to threaten absolute divorce and hopeless confusion. Now, the Anglo-Saxon can rest comfortably, without a lexicon on the table, with no wrinkles on his brow and no anguish in his heart. But in spite of the advantage of having so valuable a work in English, one must regret that the translators did not supply the missing links in several of the mathematical formulas, for, unfortunately, many of us can make no pretense to the mental agility of the mathematical acrobat. As a result, when Helmholtz gives a formula and then says "therefore," or "hence it follows," many a student, not particularly stupid in other fields, cannot see at all how "it follows." Indeed, the only way he can discover the sequence is to work it out on one or two pages of foolscap, slowly and painfully, by the sweat of his brow. If the English translators would be so merciful as to insert a few intermediate formulas, it may save some enterprising Yankee the necessity of publishing still another volume entitled *Helmholtz Made Easy*. The writer is painfully conscious of these mathematical difficulties—at least as they seemed to him when, in the later seventies, he listened to our great master in the old Physiologische Institut on the Dorotheenstrasse, during a part of one semester. Ordinarily Helmholtz was the personification of the ideal teacher. He had a fine

presence, was dignified, quiet, clear in enunciation, and logical in statement. But when he came to algebraic formulas an evil spirit seized him. The chalk would begin to fly, and the figures and signs seemed to prance over the big blackboard with such rapidity and abandon as to give a headache or epigastric pain to many a student. The simple truth is that all of the would-be students of Helmholtz are not expert mathematicians, and if the translators will recognize this, their work will be even more valuable. Dr. Southall and his collaborators have given us a magnificent piece of work, for which they deserve the everlasting appreciation of all English-speaking students of physiological optics.

LUCIEN HOWE

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*Crystals and the Fine-Structure of Matter.* By FRIEDRICH RINNE.

Translated by W. S. STILES. London: Methuen & Co., 1924.

Pp. ix+195; 11 portraits; figs. 202. 10s. 6d.

Rinne has aimed in this book to present, in a manner intelligible to the ordinary reader, a brief account of the development and present status of the science which deals with the "fine-structure" of matter. He does not confine his attention to crystals, although they naturally occupy a leading part in the discussion. Crystals, like other forms of matter, are composed of electrons, atoms, ions, radicals, and molecules, and Rinne proposes the term "leptons" as a general name for all these fine-structure particles; therefore, the theme of his book is the science of "leptology."

The book is written in a style which is accurate and lucid, without being such easy reading as Tutton's *Natural History of Crystals*; the treatment is scientific, though the subject is expressed in the simplest terms of science. Tutton is equally accurate, but has a very evident enthusiasm for his subject which is decidedly attractive.

Rinne presents the evidence to show that matter, whether crystalline or not, is essentially anisotropic, or unlike in its properties in different directions; it is only because our means of testing these properties are so gross as compared with the size of the units concerned that the anisotropy of fluids is not usually recognized, since merely average results are obtained. He discusses the transformations of matter and reaches the conclusion that these are due to discontinuous changes of energy. After treating briefly of morphotropy and isotropy, he considers crystal growth and crystal solution, which lead naturally to the effects of chemicals on crystals. Rinne pictures chemicals (and also heat) as producing some changes in crystals *before* the changes occur which are known as

chemical reactions (or transformations). These changes are certainly physical, and they may also be called chemical; it is equally clear that they are crystallographic processes.

These few illustrations may serve to show that Rinne is dealing in a fundamental fashion with a subject which is not confined to crystallography, or to chemistry, or to physics, but borrows from, and contributes to, all three of these sciences. Perhaps his new name for such a subject may gradually win recognition.

On the whole, Rinne has produced a book of very considerable importance, the simple style of which does not prevent the presentation of the latest results of research, so far as they can be generalized, and the fairly successful attempt to draw broad deductions concerning the difficult problems involved in reaching a knowledge of the ultimate nature and constitution of matter.

The translator is to be congratulated upon the highly satisfactory results of his work; he has carefully avoided rendering German constructions into their direct and awkward English equivalents. In the translation of mineral names he has not been so successful, as shown by the presence of such terms as "lead glance," "iron glance," "pyrop," and "borazite."

A. N. WINCHELL